MA 22000 Lessons 15 and 16 Notes
Section 2.2 (2nd half of text)
Some Rules of Differentiation

Using the limit definition of a derivative is tedious. Mathematicians have derived some **rules for differentiation.** In this lesson, we will begin with a few very basic rules.

I The Constant Rule

> The derivative of a constant function is zero. That is
> $\frac{d}{dx}[c] = 0,$ where c is a constant (number)

Examples:

$a)$ $\frac{d}{dx}[18] = 0$

$b)$ If $f(x) = 9,$ then $f'(x) = 0$

$c)$ If $y = \frac{7}{3},$ then $y' = 0$

$d)$ If $g(x) = \sqrt{2},$ then $\frac{dg}{dx} = 0$

II The Power Rule

> The derivative of a power is the product of the exponent and the base to one less power.
> $\frac{d}{dx}\left[x^n\right] = nx^{n-1},$ for any real number n

Examples:

$a)$ If $f(x) = x^5,$ then $f'(x) = 5x^4$

$b)$ If $y = n,$ then $y' = 1n^0$ or simply 1

$c)$ $\frac{d}{dx}\left[x^{-5}\right] = -5x^{-6}$ or $\frac{-5}{x^6}$

$d)$ If $W = r^{1/2},$ then $\frac{dW}{dr} = \frac{1}{2}r^{\left(-\frac{1}{2}\right)}$ or $\frac{1}{2\sqrt{r}}$

III The Constant Multiple Rule

If *f* is a differentiable function, then the derivative of a constant times *f* is the product of that constant and the derivative of *f*.

> If c is a constant:
>
> $$\frac{d}{dx}\left[c\,f(x)\right]=c\,f'(x)$$

Examples:

$a)$ If $y=2x^3,$ then $y'=6x^2$

$b)$ If $g(x)=\dfrac{3x^3}{5},$ then $g'(x)=\dfrac{3}{5}\left(3x^2\right)$ or $\dfrac{9}{5}x^2$

$c)$ $\dfrac{d}{dx}\left[3x\right]=3(1x^0)$ or just 3

IV The Sum and Difference Rules

> The derivative of the sum or difference of two differentiable functions is the sum or difference of their derivatives.
> $f(x)$ and $g(x)$ are two differentiable functions
>
> $$\frac{d}{dx}\left[f(x)\pm g(x)\right]=f'(x)\pm g'(x)$$

Examples:

$a)$ If $f(x)=3\sqrt{x}-4x^3,$ then $f'(x)=3\left(\dfrac{1}{2}x^{-\frac{1}{2}}\right)-4(3x^2)$

$$=\frac{3}{2\sqrt{x}}-12x^2$$

Ex 1: Find each derivative.

$a)$ $f(x)=5$ $b)$ $y=\dfrac{\pi}{2}$ $c)$ $\dfrac{d}{dx}\left[3x^{15}\right]$

$d) \quad y = 3x^4 - 5x^2$

$e) \quad \dfrac{d}{dx}\left(\dfrac{2}{x^3} - 8x + 3x^{\frac{2}{3}}\right)$

$f) \quad h(x) = 3x^{-3} + 2 - 4x$

Some functions may have to be written as a polynomial (or rewritten) before finding the derivative. Look at these examples.

Ex 2:

$a) \quad f(x) = (3x-2)^2$

$= 9x^2 - 12x + 4$

$f'(x) =$

$b) \quad y = \dfrac{4x^5 - 9x^4 - 3x + 9}{x^2}$

$= 4x^3 - 9x^2 - 3x^{-1} + 9x^{-2}$

$y' =$

Ex 3: We have learned earlier that the derivative can be used to find the slope of a tangent line to a given point.

Find the slope of the tangent line to the graph of $y = 2x^2 - 6x + \frac{7}{x}$ at the point (1, 3).

Ex 4: Find the value of the derivative of the function below at the given point.

$h(t) = 7 - \dfrac{3}{t} + 2t \qquad (3,12)$

Ex 5: Find the equation of the tangent line to the function $f(x)$ below at the given point.

$$f(x) = 3x^4 - 4x^2 + 2 \qquad (-1,1)$$

We know that the slope of a horizontal line is 0.

Ex 6: Determine any possible points at which the graph of the function below has a horizontal tangent line.

$$y = \frac{1}{6}x^3 - 2x$$