MA 22000 Notes, Lesson 36, (2nd half of text) section 4.2
Natural Exponential Functions

Summary: A general and **basic** exponential function is $f(x) = a^x$, where the base, a is any positive number except one.

Examine the function values for various bases.

A $\quad y = f(x) = 2^x$

x	-3	-2	-1	0	1	2	3
$f(x)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

B $\quad y = \left(\frac{7}{3}\right)^x$

x	-3	-2	-1	0	1	2	3
$f(x)$	$\frac{27}{343}$	$\frac{9}{49}$	$\frac{3}{7}$	1	$\frac{7}{3}$	$\frac{49}{9}$	$\frac{343}{27}$
≈	0.0787	0.1837	0.4286	1	2.3333	5.4444	12.7037

C $\quad y = f(x) = \left(\frac{2}{3}\right)^x$

x	-3	-2	-1	0	1	2	3
$f(x)$	$\frac{27}{8}$	$\frac{9}{4}$	$\frac{3}{2}$	1	$\frac{2}{3}$	$\frac{4}{9}$	$\frac{8}{27}$

D $\quad y = f(x) = (0.1)^x$

x	-3	-2	-1	0	1	2	3
$f(x)$	0.001	0.01	0.1	1	10	100	1000

A and B are examples of **exponential growth.** As x increases, $f(x)$ increases and the larger the base, the faster the function values increase. If the base of a basic exponential function is greater than 1, it will represent exponential growth.
C and D are examples of **exponential decay.** As x increases, $f(x)$ decreases and the smaller the base, the faster the function values decrease. If the base of a basic exponential function is between 0 and 1, it will represent exponential decay.

The number e is another example of an irrational number, like the number π. This number shows up a lot in the real world and especially in the financial world. The number e is approximately 2.718281828… Like π, this number is irrational; if not a terminating or repeating decimal. It is derived from the power $\left(1+\frac{1}{n}\right)^n$ as n gets very, very large. Because the number e is greater than 1, it would represent exponential growth. The number e is called the **natural base.**

Example 1: Using a basic one-line scientific calculator, find the following powers.

a) 2^{18} Press the following keys….

| 2 | then | y^x | (the power key) then | 18 | = |

You should get 262,144.

b) $\left(\frac{1}{3}\right)^9$ Press the following keys in the order shown….

| 1 | ÷ | 3 | = |

Now use the power key, then press 9 =

c) Find the following powers using your calculator. Round to 8 decimal places, if necessary.

4^{14}

$\left(\frac{8}{5}\right)^{25}$

$\left(\frac{3}{4}\right)^{12}$

$(0.3)^8$

Example 2: Using a basic one-line scientific calculator, find the following powers.

a) e^5 Press the following keys (left to right)….

| 5 | 2nd | LN |

You should get 148.41316 (rounded)

b) Find the following powers of e using your calculator. Round to 6 decimal places, if necessary.

e^5 e^2

e^1 e^{-2}

Compound Interest Formulas:

If P dollars is deposited in an account at an annual interest rate of r (in decimal form), t is number of years, and n is the number of compounding periods in one year, the following formula find the amount A in the account at the end of t years.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Note: If interest is compounded annually, n = 1.
If interest is compounded semiannually, n = 2.
If interest is compounded quarterly, n = 4.
If interest is compounded monthly, n = 12.
If interest is compounded daily, n = 365.

When the number of compounding periods n goes to infinity, we have interest compounded continuously and the formula for figuring the amount in an account with an investment of P dollars at a rate of r (in decimal form) for t years is $A = Pe^{rt}$.

Example 3:
Find the amounts in the following accounts where $2000 was invested at 8 ½ % annual interest for 15 years for each n (number of compounding periods) below. **We are finding the future value of an investment.**

$a)$ $n = 1$ (compounded annually)

$b)$ $n = 2$ (compounded semiannually)

$c)$ $n = 4$ (compounded quarterly)

$d)$ $n = 14$ (compounded monthly)

$e)$ $n = 365$ (compounded daily)

$f)$ compounded continuously

Example 4:
Which is the better investment? $5000 at 6% compounded quarterly for 12 years? Or, $5000 at 5% compounded continuously for 10 years?

Example 5:
Determine the amount of money P that should be invested at rate r to produce a final balance of $50,000 in t years. **We are finding the present value of an investment.**

$a)$ $r = 5\%,$ compounded continuously for 1 year? for 10 years? for 40 years?

$b)$ $r = 6.4\%,$ compounded daily for 1 year? for 10 years? for 20 years?