MA 220, Lesson 2 Functions & Addition/Subtraction Polynomials
Algebra: Sections 3.6 and 5.2

Definition: A relation if any set of ordered pairs. The set of first components in the ordered pairs is called the **domain** of the relation. The set of second components is called the **range** of the relation. Relations may be represented as sets, tables, diagrams, graphs, or equations.

Definition: A **function** is a relation in which no two ordered pairs have the same first components but different second components. Each element or component of the domain (input values) is paired to <u>one and only one</u> element or component of the range (output values).

Example 1: Is each relation represented a function? Give the domain and range.

a) $\{(9,81), (4,16), (5,25), (-2,4), (-6,36)\}$ Function?

Domain: Range:

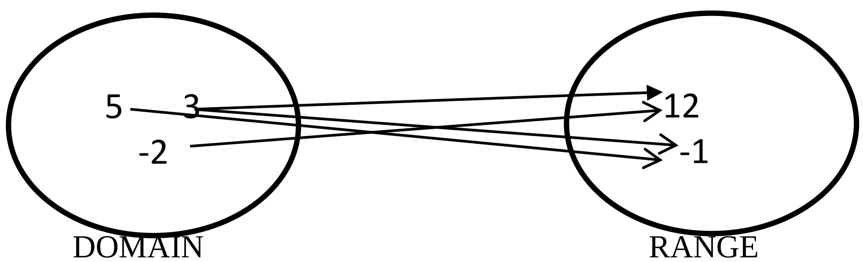
b)

x	-3	4	12	9
y	2	-1	0	2

Function?

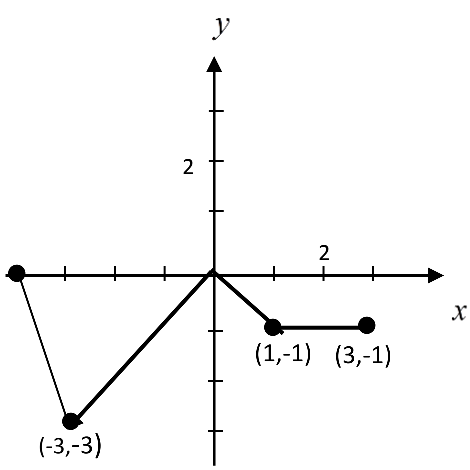
Domain: Range:

c)



Function?

d)



Function?

Domain:

Range:

e) $x = y^2$ Function? Domain: Range:

Function Notation: Functions can be 'named' by using letters. This 'name' can be used to write the function. For example; the function *h* represented by $y = 2x - x^2$ can be written as $h(x) = 2x - x^2$. This type of notation is known as **function notation**. The element of the domain (the *x*) is inside the parentheses and the result (the *y*) is the matching element of the range.

Example 2: Given the function $g(x) = |3x - 1|$, find the following function values.

$a)\quad g(-2) =$

$b)\quad g(0) =$

$c)\quad g(4) =$

$d)\quad g(\sqrt{2})$

Example 3: Given the functions $f(x) = x^2 - 2x$ and $F(x) = \dfrac{x}{3x-1}$, find…

$a)\quad f(3x) =$

$b)\quad F(a+1) =$

$c)\quad f(r+2) =$

$d)\quad F\left(\dfrac{1}{b}\right) =$

$e)\quad f(x) + f(2)$

Optional:
A function may be defined by more than one equation, such as the **piecewise-defined function** in example 4.

Example 4: Given this piecewise-defined function, find the following.

$$f(x) = \begin{cases} x+2 & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$$

$a)\quad f(0) =$

$b)\quad f(1) =$

$c)\quad f(x+2) \text{ if } x \geq 1$

In most functions the implied domain is the set of all real numbers (inputs) that yield real number values for the function values (outputs). In 'real life' problems, the domain and range are only the values that are reasonable.

Example 5: Find the domain of each function.

$a)$ $f(x) = \sqrt{2x-3}$

$b)$ $G(x) = \frac{2x}{x^2 - 3x - 4}$

$c)$ $d(t) = 50t,$ where d is the distance in miles traveled by a car in t hours

Definition: A **polynomial in a single variable** $\boldsymbol{x}$ is an algebraic expression whose terms are all of the form ax^k, where *a* (the coefficient) is any real number and *k* (the exponent) is a nonnegative integer. The **degree of the polynomial** is the greatest exponent in the polynomial and the coefficient of the term with that greatest exponent is called the **leading coefficient**. Any number term (without a variable) is called the **constant term or constant**.

Definitions: A polynomial that is written in order of descending powers of the variable is said to be in **standard form**. A polynomial with only one term is a **monomial**. A polynomial with two *unlike* terms is called a **binomial**. A polynomial with three *unlike* terms is called a **trinomial.**

Polynomial	Standard Form	Degree	Leading Coefficient
$4x - 3x^2 + 2 + x^4$	$x^4 - 3x^2 + 4x + 2$	4	1
$\frac{1}{3}x - 3x^3$	$-3x^3 + \frac{1}{3}x$	3	-3
20	20	0	20
$4a - 12a^5 + 2a^3 - 6$	$-12a^5 + 2a^3 + 4a - 6$	5	-12

To add two or more polynomials, simply combine 'like' terms. To subtract a polynomial, add the opposite (distribute the negative sign). The book demonstrates how to add or subtract horizontally or vertically. The key is to remember to distribute the negative (minus) through each term of the polynomial that is subtracted.

Example 6: Add or subtract or combine where possible in each polynomial expression.

$a)\quad (3x^2 - 2x + 9) - (5x - 2x^2 + 10)$

$b)\quad (4x + 2) - (12x - 9) - 3x + (5 - 7x)$

$c)\quad (2a^2 - 4a + 1) - [(3a^2 - a + 3) - (4a - 9a^2 + 7)]$

The **position function** gives the height above ground of an object falling to earth or hurled upward then falling back to earth. It is $h = -16t^2 + v_0 t + s_0$. It is a function of time and the height will be in feet. Velocity is in feet per second and the initial height will be in feet, time in seconds. The coefficient of t, the v_0 is the initial velocity (velocity at time 0). The s_0 will be the initial height of the object (height at time 0). Note: <u>If the initial velocity is positive, the object is projected upward; if negative, the object was projected downward; if the initial velocity is zero, the object is simply dropped.</u>

Example 7: An object has the position function $h(t) = -16t^2 + 48t + 6$. Explain what the 48 and the 6 represent. Find and interpret the following.

$a)\quad h(0)$

$b)\quad h(2)$

$c)\quad h(3)$

$d)\quad h(5)$

Example 8: The four sides of a quadrilateral can be represented by
$2a + 5,\ 3a - 1,\ 5a - 6,$ and $4a - 3.$
Find a polynomial that represents the perimeter of the quadrilateral. If $a = 12$, what is the perimeter?

Example 9: A manufacturer can produce and sell x gadgets per week. The total cost of producing these gadgets (in dollars) is given by $C(x) = 11x + 375$. The revenue from the sale of these gadgets (in dollars) is $R(x) = 26.6x$. Find a function to represent the profit from the production and sell of x gadgets per week. What is the profit from the sale of 25 gadgets?