MA 22000, Lesson 26 Notes
Section 3.1

A function is **increasing** if its function values (*y*'s) are rising as *x* values get larger. A function is **decreasing** if its function values (*y*'s) are falling as *x* values get larger.

Formal Definition:

A function *f* is increasing on an interval if for any two *x* values in that interval; $x_2 > x_1$ implies that $f(x_2) > f(x_1)$. (**as x values increase, y values increase**)
A function is decreasing on an interval if for any two *x* values in that interval; $x_2 > x_1$ implies that $f(x_2) < f(x_1)$. (**as *x* values increase, y values decrease**)

TEST for determining when a function is **increasing, decreasing, or constant** (neither increasing nor decreasing):

f is differentiable on the open interval (a, b)

1. If $f'(x) > 0$ for all x (except a possible finite number of x values) in (a,b), then f is increasing on (a,b).
2. If $f'(x) < 0$ for all x (except a possible finite number of x values) in (a,b), then f is decreasing on (a,b).
3. If $f'(x) = 0$ for all x in (a,b), then f is constant (graph horizontal) on $(a.b)$.

Critical values are the values where the derivative of a function is either zero or undefined. Only at these critical values can the derivative 'change signs' or change from increasing to decreasing or decreasing to increasing.

Guidelines for finding Intervals of Increasing or Decreasing for a function *f*:

1. Find the derivative of *f*.
2. Find the critical values by finding all *x* for which the derivative is 0 or undefined. Use these values to determine test intervals and make a **sign diagram or sign chart.**
3. Test a number for each interval in the sign chart and determine the sign of the result.
4. Conclude whether the function is increasing or decreasing in that interval according to the sign.

Example 1: Evaluate the derivative of the function at the given point(s).

$$f(x) = (x+2)^{2/3} \qquad \text{points: } (-3,1), (-1,1), (-2,0)$$

Examples: Determine the intervals of increasing and decreasing for each function. Use a sign chart.

Ex 2: $f(x) = -x^2 - 2x - 1$

Ex 2: $p(x) = \frac{2}{3}x^3 - x^2$

Ex 3: $g(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + 2$

Ex 4: $y = x^4 + 8x^3 + 18x^2 - 8$

Ex 5: The profit P made by a software company from sales of x units of a software package (all of the same price) can be modeled by $P = 3.65x - \frac{x^2}{4000},\ 0 \le x \le 1{,}000{,}000$.

Find the intervals on which P is increasing and decreasing.
How many units should be sold to maximize profit?