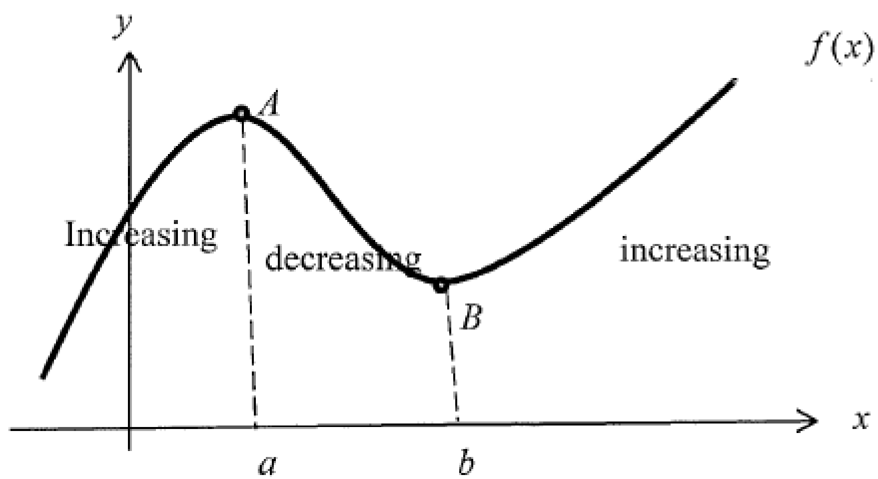
MA 22000 Lesson 27 Notes
(2nd half of text, section 3.2)
Relative Extrema, Absolute Extrema in an interval

In the last lesson, we found intervals hwere a function was increasing or intervals where that function was decreasing. At a point, where a function changes from increasing to decreasing, there will be a **relative maximum** (a function value larger than all other function values close by). At a point, where a function changes from decreasing to increasing, there will be a **relative minimum** (a function value smaller than all other function values close by). Examine the picture below.



******Important******

At $x = a$ there is relative maximum. (Notice this point is 'higher' than all other points close by.) If point A has coordinates (2, 7), we would say there is a relative maximum of 7 and it occurs when $x = 2$; or we could say the relative maximum is $f(2) = 7$.

At x = b there is a relative minimum. (Notice this point is 'lower' than all other points close by.) If point B has coordinates (5, 3), we would say there is a relative minimum of 3 and it occurs at $x = 5$; or we could say the relative minimum is $f(5) = 3$.

*****Relative extrema (relative minimums or relative maximums) will only occur at critical values for the function. That is to say, they only occur when the derivative of the function equals zero or is undefined.*****

Sign diagrams that were used to find intervals of increasing and decreasing can also be used to find relative extrema. If f is a continuous function on the open interval (a, b) that contains c, a critical value; then $f(c)$ can be classified as a relative minimum, a relative maximum, or neither using the following guidelines.

1. If the derivative is negative (decreasing) to the left of $x = c$ and positive (increasing) to the right of $x = c$, then $f(c)$ is a relative minimum.
2. If the derivative is positive (increasing) to the left of $x = c$ and negative (decreasing) to the right of $x = c$, then $f(c)$ is a relative maximum.
3. If the derivative has the same sign to the left and right of $x = c$, then $f(c)$ is neither a relative minimum or a relative maximum of the function f.

Example 1: Use a sign diagram to determine any relative minimum(s) or relative maximum(s) of this function.

$$f(x) = x^2 + 7x - 18$$

Example 2: Use a sign diagram to determine any relative extrema of this function. Label your answer(s).

$$g(x) = -x^3 - 12x^2 - 48x - 64$$

Example 3: Use a sign diagram to determine any relative extrema of this function. Label your answer(s).

$$y = \frac{1}{5}x^5 - x$$

Example 4: Use a sign diagram to determine any relative extrema of this function. Label your answer(s).

$$y = x^4 - 2x^3$$

Example 5: Use a sign diagram to determine any relative extrema of this function. Labsl your answer(s).

$$h(x) = x + \frac{1}{x}$$

Absolute Extrema: In a closed interval of a function, there is one point (or more) point(s) that will have a greater function value (be higher) than all others in that interval. That function value is called the **absolute maximum or simply maximum value.** Likewise, in a closed interval of a function, where will be one point (or more) that will have a lesser function value (be lower) than all others in that interval. That function value is called the **absolute minimum or simply minimum value.**

Definition of Absolute Extrema:

Let f be a function defined on an interval containing a value c.

1. $f(c)$ is an absolute minimum of f on that interval if $f(c) \le f(x)$ for every x in f.
2. $f(c)$ is an absolute maximum of f on that interval if $f(c) \ge f(x)$ for every x in f.

Absolute extrema in a closed interval can occur at endpoints of the interval or at any relative extrema of that interval.

<u>Guidelines for finding Absolute Extrema on a Closed Interval:</u> To find the absolute extrema a continuous function on a closed interval [a, b]:

1. Find the function values at each of the critical values from the interval (a, b).
2. Find the function values at each endpoint (a and b) of the interval.
3. The least of these numbers is the absolute minimum or minimum and the greatest is the absolute maximum or maximum. They occur at a specified x value.

Example 6: Find the minimum and maximum values of $f(x) = x^2 - 8x + 10$ on the interval [0, 7].

Example 7: Find the absolute extrema of the function $f(x) = x^3 - 3x^2$ on $[-1,1]$.

Example 8: Find the absolute extrema of $h(x) = \frac{1}{3-x}$ on $\left[0, \frac{11}{4}\right]$.

Example 9: Find the absolute extrema of $f(x) = (x-1)^{2/3}$ on [-7, 2].

Example 10: Applied Problem

A retailer has determined the cost *C* for ordering and storing *x* units of a product to be modeled by $C = 3x + \frac{30000}{x}$ $1 \le x \le 200$. (The delivery truck can bring at most 200 units per order.)

Find the order size that will minimize the cost.