MA 220 Lesson 28 Notes
Section 3.3 (p. 191, 2nd half of text)

The property of the graph of a function 'curving' upward or downward is defined as the <u>concavity</u> of the graph of a function.

Concavity if how the derivative (that describes if a function is decreasing or increasing) is changing.

Remember that the first derivative is the slope of a tangent line to a curve. Suppose the tangent lines to a curve are drawn as x gets larger. What happens to the slopes of the tangent lines?

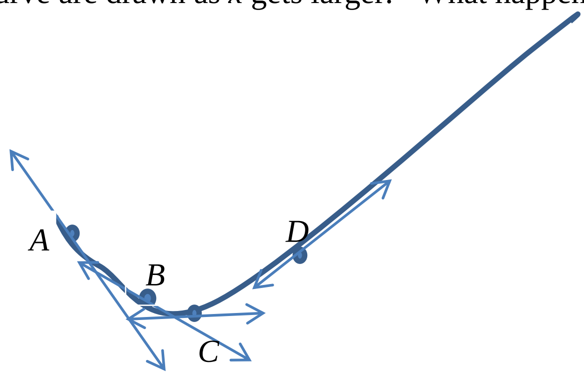


FIGURE 1

The slope of the tangent line at point A is negative. At point B (which is greater than A), the slope is negative, but a larger numeric value. At point C, the slope is almost 0 (larger than the slopes at points A and B). At point D the slope of the tangent line is positive. Conclusion: As x gets larger (A to B to C to D, left to right), the slopes of the tangent lines are getting larger. The change in the derivative is increasing.

The curve in figure 1 is **concave upward. Notice the curve lies above its tangent lines.**

Suppose the tangent lines to the curve below are drawn as x gets larger.

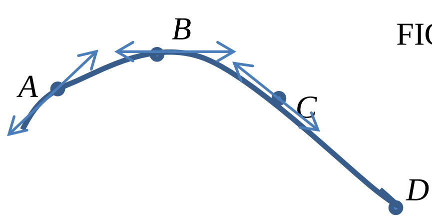


FIGURE 2

The slope of the tangent line at point A is a positive number. At point B is slope is still positive, but much smaller (almost zero). By point C, the slope is negative and at point D, the slope of the tangent line is an even smaller negative number. Conclusion: As x gets larger (A to B to C to D, left to right), the slopes of the tangent lines are getting smaller. The change in the derivative is decreasing.

The curve shown in figure 2 is **concave downward**. **Notice the curve lies below its tangent lines.**

A graph of a function in an interval is considered 'concave upward' if the *first derivative of the function is increasing* on the interval. A graph of a function in an interval is considered 'concave upward' if the *first derivative of the function is decreasing* on the interval. (Also look at Figure 3.20, p. 191 of text.)

In other words, the second derivative of a function on a given interval can determine if the function is concave upward or concave downward in that interval.

Test for concavity:
f is a function whose 2nd derivative exists on an interval:

1. If $f''(x) > 0$ for all *x* in the interval, then the function *f* is concave upward on that interval.
2. If $f''(x) < 0$ for all *x* in the interval, then the function *f* is concave downward on that interval.

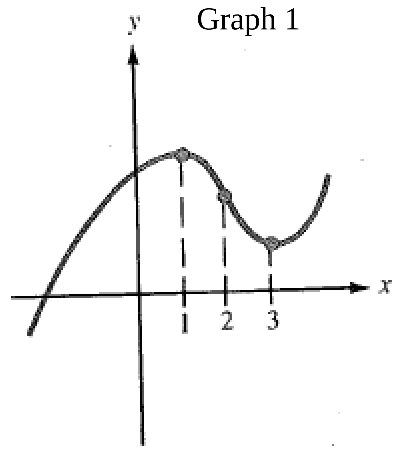
Using a 2nd derivative sign chart to determine intervals of concavity.

1. Find the values of *x* where the second derivative of a function is zero or undefined.
2. Use the *x*-values to determine intervals to use in a sign chart.
3. Select a 'test' value in each interval and determine the sign of the second derivative. If that value is positive, the function is concave upward in that interval; negative, the function is concave downward in the interval.

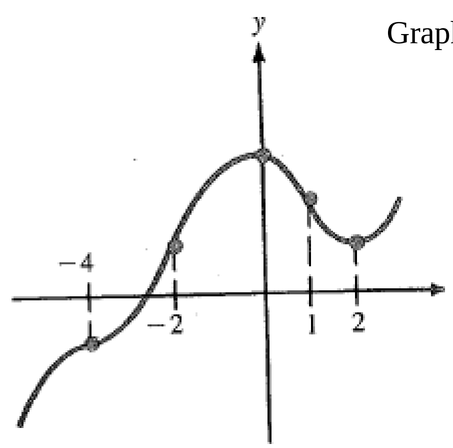
Definition of a **Point of Inflection**: If a graph of a continuous function has a tangent line at a point where the concavity changes from upward to downward (or downward to upward), then that point is a **point of inflection.** Figure 3.25 on page 195 of the textbook (2nd half) is a good illustration of two points of inflection.

Example 1: For each graph, for points marked at certain *x* values, determine if the second derivative of the function would be positive, negative, or if the point would be an inflection point.

Graph 1



Graph 2



$x = 1$:

$x = 2$:

$x = 3$:

$x = -4$:

$x = -2$:

$x = 0$:

$x = 1$:

$x = 2$:

Example 2: Find any intervals where the graph of $f(x) = x^3 - 3x^2 + 3x + 1$ is concave upward and any intervals where the graph is concave downward and any point(s) of inflection.

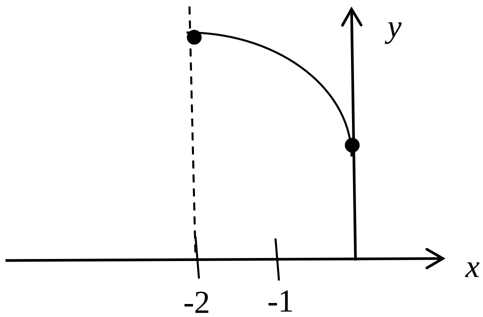
Example 3: Find any intervals where the graph of $g(x) = -\frac{1}{6}x^4 + \frac{1}{3}x^3 + 2x^2 + 3x$ is concave upward and any intervals where the graph is concave downward and any point(s) of inflection.

Example 4: Find any intervals where the graph of $y = \frac{x^2}{x-3}$ is concave upward and any intervals where the graph is concave downward and any point(s) of inflection.

Example 5: Find any intervals where the graph of $h(x) = (x^2 - 5)^3$ is concave upward and any intervals where the graph is concave downward and any point(s) of inflection.

Example 6: Look at each graph of a function *f*. Determine the signs of $f'(x)$ and $f''(x)$.

a)



b)