MA 22000 Lesson 39 Lesson Notes
(2^{nd} half of text) Section 4.4, Logarithmic Functions

Definition of General Logarithmic Function:

A logarithmic function, denoted by $y = \log_b x$, is equivalent to $b^y = x$.

In previous algebra classes, you may have often used the number 10 as a base for a logarithmic function. These were called common logarithms. In calculus, the most useful base for logarithms is the number *e*. These are called **natural logarithms.**

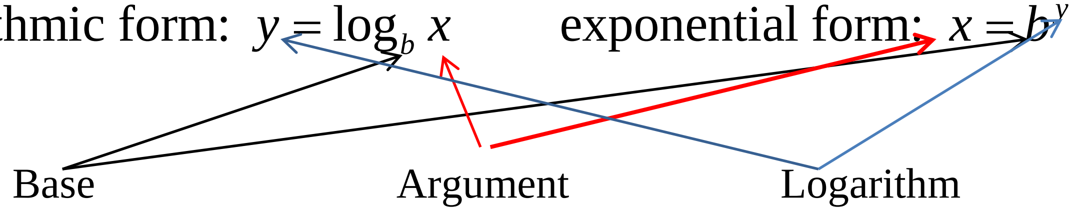
Definition of the Natural Logarithmic Function:

The natural logarithmic function is denoted by $y = \ln x$, which is equivalent to saying $y = \log_e x$. The function $y = \ln x$ is true if and only if $e^y = x$. The natural logarithmic function is the inverse of the natural exponential function.

The Natural Logarithmic function can be written in logarithmic form or exponential form. Examine the comparison below.

$$y = \log_b x \Leftrightarrow b^y = x$$

logarithmic form: $y = \log_b x$ exponential form: $x = b^y$



$$y = \ln x \Leftrightarrow e^y = x$$

logarithmix form: $y = \ln x$ exponential form: $x = e^y$

(For the natural logarithmic function: the *x* is called the **argument,** the *y* is called the **logarithm**, and the base is the number *e* .)

Example 1: Convert each logarithmic form to exponential form or each exponential form to logarithmic form.

1) $3^4 = 81 \rightarrow$

2) $25^{\frac{1}{2}} = 5 \rightarrow$

3) $8^{-2} = \dfrac{1}{64} \rightarrow$

4) $m^{rp} = (x+4) \rightarrow$

5) $(2a)^{y+7} = p \rightarrow$

6) $e^m = (x+1) \rightarrow$

7) $\log_2 32 = 5 \rightarrow$

8) $\log_{\frac{1}{2}}\left(\dfrac{1}{8}\right) = 3 \rightarrow$

9) $\log_5 \sqrt{5} = \dfrac{1}{2} \rightarrow$

10) $\log_q(2mn) = 12 \rightarrow$

11) $\log_{(x+3)} 200 = rs \rightarrow$

12) $\ln(3x) = u \rightarrow$

Properties of Logarithmic Functions

A Inverse Properties: $\ln e^x = x$ $\quad e^{\ln x} = x$

(based on inverse property: $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$)

B Product Property: $\ln xy = \ln x + \ln y$

(based on rule for multiplying powers: $b^x \cdot b^y = b^{x+y}$)

C Quotient Property: $\ln\frac{x}{y} = \ln x - \ln y$

(based on rule for dividing powers: $\frac{b^x}{b^y} = b^{x-y}$)

D Power Property: $\ln x^n = n\ln x$

(based on the rule for a power to a power: $\left(e^y\right)^n = e^{ny}$)

*****There is no general property that can be used to rewrite $\ln(x+y)$ or $\ln(x-y)$. They are not $\ln x + \ln y$ or $\ln x - \ln y$.) See the comment at the bottom of page 283.

Rewriting a logarithm of a single quantity as the sum, difference, or multiple of logarithms is called *expanding* the logarithmic expression. The reverse procedure is called *condensing* a logarithmic expression.

Use the properties A – D to write the expression as a sum, difference, or multiple of logarithms (expand the logarithm).

13) $\ln(7r)$

14) $\ln(2x^2y)$

15) $\ln(20e^5)$

16) $\ln\left(\frac{9}{y}\right)$

17) $\ln\left(\frac{x}{5}\right)$

18) $\ln\sqrt{y}$

19) $\ln\left(\frac{xy}{\sqrt{z}}\right) =$

20) $\ln(e^4 x^2 \sqrt[3]{y})$

21) $\ln\sqrt{\frac{x^2}{x+2}}$

Use the properties A – D to write the expression as the logarithm of a single quantity (condense the logarithmic expression.)

19) $\ln a - \ln(a+3)$

20) $\ln 3 + 2\ln y - \ln x - \frac{1}{2}\ln(x+2)$

21) $\frac{1}{3}\ln(x-2) + 2\ln x - 2\ln 4$

Solve each equation for *x*. Give exact answer and approximate answer to 4 decimal places.

22) $3\ln x = 12$

23) $2e^{-0.3x} - 3 = 5$

24) $2e^{4-x} = 12$