MA 22000 Lesson 40 Notes
Section 4.5 (part 1)

Derivative of the natural logarithmic function:

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ (In words, the derivative of a natural logarithmic function is the reciprocal of the argument.)

Let u be a function of x, then
$\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot u'$ or $\frac{1}{u} \cdot \frac{du}{dx}$ (in words, the reciprocal of the argument time derivative of argument)

Example 1: Find the derivative of each.

$a)$ $f(x) = \ln 6x$ $\qquad$ $b)$ $g(x) = \ln(x^2 - 4)$

$c)$ $y = x^3(\ln x)$

Sometimes it is easier to rewrite the function (using the properties of logarithms) prior to finding the derivative. Examine the example below.

Find the derivative of $f(x) = \ln\sqrt{2x+3}$

$$f(x) = \ln(2x+3)^{1/2}$$

Rewrite the function:

$$f(x) = \frac{1}{2}\ln(2x+3)$$

outer function: $\frac{1}{2}\ln u$ $\qquad$ inner function: $u = 2x+3$

$$f'(x) = \text{(dee outer)(dee inner)}$$

$$= \left(\frac{1}{2} \cdot \frac{1}{u}\right)(2) = \frac{1}{u} = \frac{1}{2x+3}$$

Example 2: Find the derivative of each by re-writing first using the properties of logarithms.

$a)$ $y = \ln \sqrt[3]{x+2}$

$b)$ $g(x) = \ln[x^2 \sqrt{x+1}]$

$c)$ $y = \frac{1}{2}(\ln x)^6$

$d)$ $y = \ln\left(\frac{x^2}{x+1}\right)$

$e)$ $y = \ln(x\sqrt{4+x^2})$

Example 3:
Find the slope of the graph at the indicated point. Then, find the equation of the tangent line at the point.

$a)\quad f(x) = 2x(\ln x) \qquad (1,0)$

$b)\quad y = \dfrac{\ln x}{x} \qquad \left(e, \dfrac{1}{e}\right)$

$c)\quad g(x) = \ln\left(\dfrac{2(x+1)}{x}\right) \qquad (-2,0)$