MA 22000 Lesson 42 Notes
Section 4.6, Exponential Growth and Decay

Quote from textbook (page 299): "Real-life situations that involve exponential growth and decay deal with a substance or population whose **rate of change at any time *t* is proportional to the amount of the substance present at that time.**"

This equation describes the statement above. The value of the constant k is the growth or decay rate.

$$\frac{dy}{dt} = ky$$

<u>Law of Exponential Growth and Decay</u>: If y is a positive quantity whose rate of change with respect to time is proportional to the quantity present at any time t, then y is of the form

$$y = Ce^{kt}$$

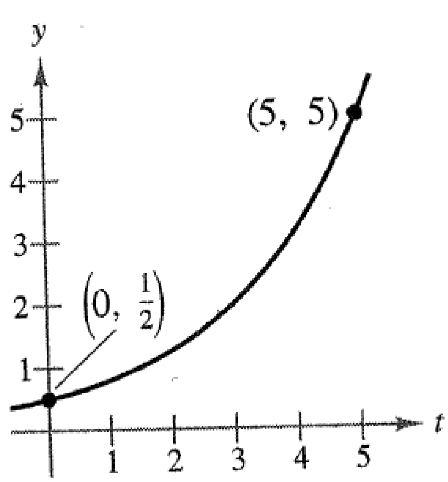
where C is the initial value or initial amount and k is the constant of proportionality. Exponential growth is indicated by $k > 0$ and exponential decay by $k < 0$.

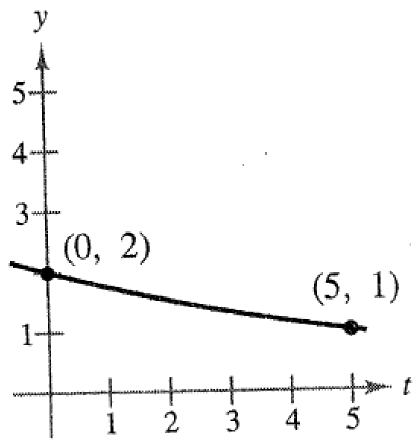
Guidelines for Modeling Exponential Growth and Decay

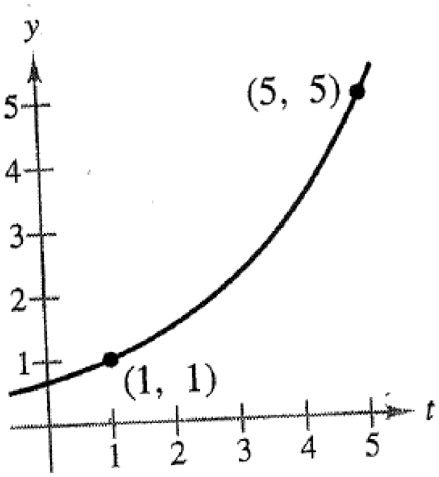
1. Use the given information to write two sets of conditions involving y and t (two ordered pairs).
2. Substitute the given conditions into the model $y = Ce^{kt}$ and use the results to solve for the constants C and k. (Note: If one of the conditions involves time $t = 0$, you know the value for C (the initial amount).)
3. Once both constants have been determined, the model $y = Ce^{kt}$ can be used to answer any questions.

Example 1: Find the exponential function $y = Ce^{kt}$ that passes through the two points of the each graph below.

a)



b) 

c) 

Example 2: Use the given information to identify the growth or decay rate *k*, and then write the equation $y = Ce^{kt}$.

a) $\frac{dy}{dt} = 3y, \quad y = 5$ when $t = 0$

b) $\frac{dy}{dt} = -\frac{3}{2}y, \quad y = 30$ when $t = 0$

Applied Problems:

Example 3:

The number of a certain type of bacteria increases continuously at a rate proportional to the number present. There are 250 present at time t and 400 present 3 hours later.

a) How many will there be 5 hours after the initial time?

b) How long will it take for the population to triple?

Example 4:

In 1990, the population of a small city was 4.5 thousand. By 2000, the population has risen to 7.8 thousand. Assume the population can be modeled by exponential growth.

a) Estimate the populations of the city in 2005, 2010, and 2015.

b) How many years from 1990 until the population is double the 2010 number?