

**Functions:**

A **function** is a relationship between two variables such that to each value of the independent variable there corresponds exactly one value of the dependent variable.

The **domain** of the function is the set of all values of the independent variable (the  $x$ ) for which the function is defined (the  $x$  values). The **range** of the function is the set of all values of the dependent variables (the  $y$ ). The domain could also be described as the set of ‘inputs’ and the range could be described as the set of ‘outputs’.

**Function Notation:**

When an equation of a function is solved for the ‘dependent variable’ or the  $y$ , we can write the function by giving it a name. This is known as function notation. For example, the function  $y = 2x^2 - 3x + 1$ , could be given the ‘name’  $f$  and written  $f(x) = 2x^2 - 3x + 1$ . The function  $n = \sqrt{3a + 4}$  could be given the name  $r$  and written  $r(a) = \sqrt{3a + 4}$ . The function value, usually written  $f(x)$ , etc., is the value of the dependent variable when  $x$  is the value given.

For example, using the above functions  $f$  and  $r$ ...

$$\begin{aligned} f(-3) &= 2(-3)^2 - 3(-3) + 1 \\ &= 2(9) - 3(-2) + 1 \\ &= 18 + 6 + 1 \\ &= 25 \end{aligned}$$

We call these values (the 25 and the 5) the **function values**.

$$\begin{aligned} r(7) &= \sqrt{3(7) + 4} \\ &= \sqrt{21 + 4} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

**When finding a function value, you are asked to evaluate the function for one or more given independent value(s).**

Example 1: Find the following function values (evaluate the function for each given value).

$$f(x) = 2x - 5 \quad g(x) = \frac{3x + 2}{x - 3} \quad h(x) = \sqrt{5x - 1} \quad r(x) = 2x^2 - 7x + 3$$

a)  $f(12)$

b)  $f(a)$

c)  $f(x-1)$

d)  $g(8)$

e)  $g(n), n \neq 3$

f)  $g(5k), k \neq \frac{3}{5}$

g)  $h(4)$

h)  $h(q), q \geq \frac{1}{5}$

i)  $h(n+2), n \geq -\frac{9}{5}$

j)  $r(-3)$

k)  $r(t)$

l)  $r(x-4)$

Given a function  $f(x)$ , there is a ratio that has a special significance in calculus. It is known as the **difference quotient**. **The difference quotient for a function  $f(x)$  is...**

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Ex 2: Find the value of the difference quotient for function  $r$  of example 1.

### **Combinations of Functions:**

**Definition:** The function given by  $(f \circ g)(x) = f(g(x))$  is the **composite of  $f$  with  $g$** . (It is usually read  $f$  circle  $g$  of  $x$  or  $f$  of  $g(x)$ .) Note:  $(f \circ g)(x)$  does not necessarily equal  $(g \circ f)(x)$ .

Ex 3: For the functions  $f(x) = 2x^2 - 4x$  and  $g(x) = 3x - 5$ , find and simplify (a)  $(f \circ g)(x)$  and (b)  $(g \circ f)(x)$ .

Ex 4: for the functions  $h(u) = \frac{1}{u^2}$  and  $k(u) = \frac{u}{u+1}$ , find and simplify (a)  $(h \circ k)(u)$  and (b)  $(k \circ h)(u)$ .