MA 22000 Lesson 31 Notes
Section 3.4 Optimization Problems

An optimization problems involves finding a value that would determine a maximum or minimum for a problem.

For many of these problems, you will have to write two equations initially. The primary equation describes the quantity to be optimized (maximized or minimized.) The secondary equation will help express the primary equation as a function of only one variable.

Guidelines for Solving Optimization Problems:

1) Identify all given information and all quantities to be determined. You might need to sketch a diagram or picture.

2) Write a primary equation for the quantity that is to be maximized or minimized.

3) Reduce the primary equation to one having a single variable. This may involve the use of a secondary equation.

4) Determine a reasonable domain for the primary equation; values that 'make sense'.

5) Find the first derivative of the primary equation to find critical values. Use the techniques discussed in class to determine the value of the variable that gives the maximum or minimum (usually in a closed interval) and what that maximum or minimum value is.

Ex 1: Two positive numbers have a sum of 70. Find the pair that has a **maximum product.**

Let x = first positive and y = second positive number

Primary equation: Product = (1^{st})(2^{nd})

$$P = xy$$

Secondary equation: Sum = 1^{st} + 2^{nd}

$$70 = x + y \rightarrow y = 70 - x$$

Rewrite Primary equation by substitution from secondary equation:

$$P = x(70 - x)$$

$$P = 70x - x^2$$

A reasonable domain is (0, 70).

$P' = 70 - 2x$

$2(35 - x) = 0$

$x = 35$ is the critical value.

	$(0,35)$	$(35,70)$
2	+	+
$35 - x$	+	−
Result	+	−
	INC	DEC

The sign chart at the right shows there is a relative maximum when x = 35,

x = 35
y = 70 -35 = 35
The two positive numbers are 35 and 35 and the maximum product is 1225..

Ex 2: Find two numbers those product is a minimum and whose difference is 4.

Let x = larger number and y = smaller number

Primary Equation: Product = (larger)(smaller) or $P = xy$
Secondary equation: $x - y = 4$ or $x - 4 = y$

Rewrite Primary equation: $P(x) = x(x-4)$, $P(x) = x^2 - 4x$

derivative: $P'(x) = 2x - 4$, $2(x-2) = 0$

2 is a critical value

	$(-\infty, 2)$	$(2, \infty)$
2	+	+
$x - 2$	−	+
Result	−	+
	DEC	*INC*

The sign chart at the left shows a minimum when x is 2.

$y = 2 - 4 = -2$

Numbers, whose difference is 4, with a minimum product are 2 and -2.

Ex 3: The product of two numbers is 72. Minimize the sum of the second number and twice the first number.

Example 4: A lifeguard needs to rope off a rectangular swimming are in front of a beach, using 240 yards of rope and floats. What dimensions of the rectangle will maximize the area. (Note: The shoreline is one side of the rectangle.) What is the maximum area?

Example 5: A carpenter is building a rectangular shed with a fixed perimeter of 54 feet. What are the dimensions of the largest shed that can be built and what is its area?

Example 6: A farmer wants to enclose two rectangular areas near a river, one for sheep and one for horses. There are 480 yards of fencing available. What is the largest total area that can be enclosed?