Summer MA 15200 Lesson 10 Appendix D, Section 1.4

If person or machine *A* can complete a job in *a* hours and person or machine *B* can complete the same job in *b* hours, the part of the job each does in one hour is represented by $\frac{1}{a}$ and $\frac{1}{b}$. If both work at the same time (*x* hours) until the job is done, the work is represented by $\frac{1}{a}(x)+\frac{1}{b}(x)=1\text{ job}$. In other words (rate)(time) + (rate)(time) = 1 job

Ex 1: An old computer can do the weekly payroll in 5 hours. A newer computer can do the same payroll in 3 hours. How long will it take both computers working together to finish the job? (Round to the nearest tenth of an hour, if necessary.)

Ex 2: One pump can fill a gasoline storage tank in 8 hours. With a second pump working simultaneously, the tank can be filled in 3 hours. How long would it take the second pump to fill the tank operating alone?

Ex 3: Lynn can mow a large community lawn in 2 hours alone and Lisa can mow the same lawn with the same type of mower in 3 hours alone. Lynn has already been mowing for 1 hour when Lisa joins her. How long would it take them to finish mowing the lawn?

Ex 4: A conservationist is trying to estimate the number of fish in a small lake. He removes 100 fish from the lake and tags them. Three weeks later, he returns and collects 500 fish, 25 which had been previously tagged. Approximately, how many fish are in the lake?

I The imaginary unit *i*

Even though it does not seem logical to speak of square roots of negative numbers, they are important and play a role in some algebra and science fields.

A number *i* is defined to be the solution of the equation $x^2 = -1$. Therefore ...

The **imaginary unit *i*** is defined by $i = \sqrt{-1}$, where $i^2 = -1$.

A square root of a negative number can be represented using *i* in the following manner.
$\sqrt{-36} = \sqrt{36} \cdot \sqrt{-1} = 6i$

<u>Ex 5:</u> Write each number using the imaginary unit.

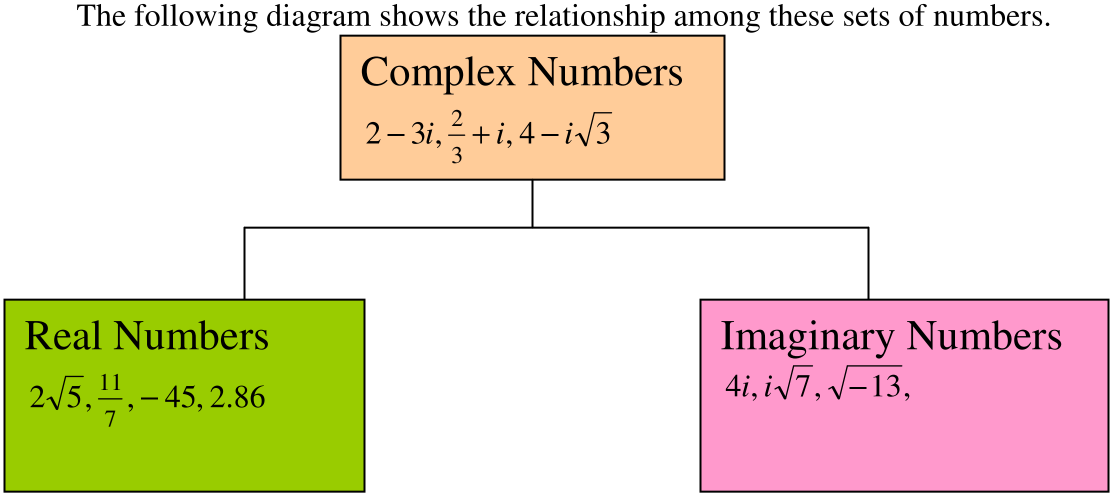
$a)\ \sqrt{-100}$

$b)\ \sqrt{-\frac{49}{4}}$

$c)\ \sqrt{-18}$

II The Set of Complex Numbers

A new set of numbers is created using imaginary numbers. A **complex number** is formed by adding a real number to an imaginary number.
The following diagram shows the relationship among these sets of numbers.



A complex number is written in *a* + *bi* form (standard form), where *a* is the 'real part' and *bi* is the 'imaginary part'.
If *a* = 0 (0+ *bi*), the number is a pure imaginary number.
If *b* = 0, (*a* + 0i), the number is a pure real number.
Therefore, every real number can be written as a complex number and every imaginary number can be written as a complex number.

If b is contains a radical, we usually write the i before the radical. These are examples of complex numbers: $2-3i,\ \ 4+\frac{2}{3}i,\ \ -8,\ \ 5i,\ \ 6+i\sqrt{2},\ \ \pi-3i\sqrt{5}$

III Operations with Complex Numbers

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$
$$(a+bi)-(c+di)=(a-c)+(b-d)i$$

Basically, combine like terms.

Ex 6: Add or subtract and write in standard form.

$a)\quad (6-2i)+(3+14i)=$

$b)\quad \left(\frac{3}{4}-3i\right)-\left(\frac{2}{3}+7i\right)=$

$c)\quad (14-\sqrt{-16})-(\sqrt{-81}-2)=$

$d)\quad (3i+12)+(17-\sqrt{-49})=$

$e)\quad (4-2i)-(6+3i)+(12-2i)=$

Note: You must change the square roots of negative numbers to pure imaginary number before adding or subtracting.

IV Multiplication of Complex Numbers

$$(a+bi)(c+di)=ac+adi+bci+bdi^2$$
$$=ac+adi+bci-bd\ \ (i^2=-1)$$
$$=(ac-bd)+(ad+bc)i$$

Basically, use FOIL when multiplying and remember to simplify $i^2=-1$ and combine 'like' terms.

Ex 7: Find each product in standard form.

$a)\quad (4-5i)(2-6i)=$

$b)\quad (4+\sqrt{-9})(-2-\sqrt{-36})=$

$c)\quad (4-7i)^2=$

$d)\quad (6-i\sqrt{2})(6+i\sqrt{2})$

$e)\quad 4i\left(\frac{3}{4}-\frac{1}{4}i\right)$

V Complex Conjugates and Division of Complex Numbers

Remember the conjugate of $a+bi$ is $a-bi$ and vice-versa. The product of conjugates is a^2-b^2. Because the square of a real number is always a rational number, the product of two conjugates will be rational (without a radical). This product will help us divide complex numbers. Multiply numerator and denominator by the conjugate of the denominator, just like you did when rationalizing a fraction with a binomial denominator containing a radical.

Ex 8: Divide and write answer in standard form.

$a)\quad \frac{12}{3i}=$

$b) \quad \dfrac{-3}{2-4i} =$

$c) \quad \dfrac{4+2i}{3-i} =$

$d) \quad \dfrac{12+\sqrt{-9}}{-3-\sqrt{-16}}$

<u>Ex 9:</u> Perform the operations and write in standard form.

$a) \quad \sqrt{-121} - \sqrt{-196} =$

$b) \quad \left(\sqrt{-100}\right)\left(\sqrt{-81}\right) =$

$c) \quad (2-\sqrt{-7})^2$

$d) \quad \dfrac{-15-\sqrt{-75}}{10}$

<u>Ex 10:</u> Complex numbers are used in electronics to describe the current in an electric circuit. Ohm's law relates the current, *I*, in amperes, the voltage, *E*, in volts and the resistance, *R*, in ohms by the formula $E = IR$. If the voltage is 6 - 2*i* volts and the resistance is 5 + *i* ohms, find the current.

VI (Optional) Powers of the Imaginary Unit

There is a pattern in the powers of the imaginary unit, *i*.

$i^0 = 1$	$i^4 = i^2 i^2 = (-1)(-1) = 1$	$i^8 = 1$
$i^1 = i$	$i^5 = i^4 i = 1i = i$	$i^9 = i$
$i^2 = -1$	$i^6 = i^4 i^2 = (1)(-1) = -1$	$i^{10} = -1$
$i^3 = i^2 i = -1i = -i$	$i^7 = i^4 i^3 = 1(-i) = -i$	$i^{11} = -i$

Notice: Even powers of *i* are either 1 or -1 and odd powers of *i* are either *i* or -*i*.

To evaluate a power of *i*: i^n

1. Determine how many groups of 4 are in *n* by dividing *n* by 4.
2. The power can be written as $i^n = i^r$ where *r* is the remainder after dividing.
3. Simplify i^r. The answer will be either 1, -1, *i*, or -*i*.

<u>Ex 11:</u> Evaluate each power.

$a)$ $i^{49} =$

$b)$ $i^{256} =$