Summer MA 15200 Lesson 11 Section 1.5 (part 1)

The **general form** of a **quadratic equation** is $ax^2 + bx + c = 0$, where *a, b,* and *c* are real numbers and $a \neq 0$. This is a second degree equation.

There are four ways to possibly solve quadratic equations.
1. **Solving by Factoring**
2. **Solving by the Square Root Property**
3. **Solving by Completing the Square**
4. **Solving with the Quadratic Formula**

I Solving by Factoring

Zero-Product Principle: If the product of two algebraic expressions is zero, then at least one of the factors is equal to zero. If $AB = 0$, then $A = 0$ or $B = 0$.

Steps:
1. If necessary, rewrite the equation in general form (zero on one side, polynomial in descending order).
2. Factor the polynomial completely.
3. Apply the zero-product principle, setting each factor containing a variable equal to zero.
4. Solve the resulting equations.

Ex 1: Solve by using factoring.

$a)$ $x^2 = 28 - 3x$

$b)$ $x^2 - 121 = 0$

$c)$ $3x^2 + 30x = 0$

$d)$ $5x + 12x^2 = 2$

Note: If a quadratic equation can be solved by factoring, the solution(s) is(are) rational numbers. If the equation cannot be solved by factoring, the solutions are conjugate irrational numbers or conjugate complex numbers.

II Solving by the Square Root Property

Square Root Property:

If the quadratic equation can be written in the form $u^2 = d$, where u is a variable or an expression with a variable and d is nonzero real number, then the solutions can be found using the equations $u = \sqrt{d}$ and $u = -\sqrt{d}$.

Ex 2: Use the square root property to solve each equation.

$a)$ $(r-1)^2 = 25$

$b)$ $(2y-5)^2 - 49 = 0$

$c)$ $9x^2 + 6x + 1 = 100$

$d)$ $8x^2 + 50 = 0$

$e)$ $(n-2)^2 = -81$

III Solving by Completing the Square

A perfect square trinomial with a leading coefficient of 1 has the form $x^2 + 2bx + b^2$. Notice that half of the middle coefficient square equals the last term. $\left[\frac{1}{2}(2b)\right]^2 = b^2$ If the leading coefficient is a 1, every binomial of the form $x^2 + bx$ can be made into a perfect square trinomial by adding $\left(\frac{1}{2}b\right)^2$. If this value is added to both sides of an equation of the form $x^2 \pm bx = c$, then the square root property can be used to solve the equation. This process is called the **completing the square** process.

Here is an example:

$$x^2 + 10x + 24 = 0$$
$$x^2 + 10x = -24$$
$$x^2 + 10x + \left(\frac{1}{2}\cdot 10\right)^2 = -24 + \left(\frac{1}{2}\cdot 10\right)^2$$
$$x^2 + 10x + 25 = -24 + 25$$
$$(x+5)^2 = 1$$
$$x + 5 = \sqrt{1} \quad \text{and} \quad x + 5 = -\sqrt{1}$$
$$x + 5 = 1 \qquad x + 5 = -1$$
$$x = -4 \qquad x = -6$$

*There is a good visual picture of completing the square on page 141 of the textbook.

Ex 3: What number should be added to each binomial so that it becomes a perfect square trinomial?

$a)\quad x^2 + 14x$

$b)\quad r^2 - 22r$

$c)\quad n^2 + 3n$

Any quadratic equation can be solved using completing the square if each term is divided by the leading coefficient. However, it is recommended you only use this method when the leading coefficient is already a 1.

Here are the steps.

1. Use this procedure only on equations where the leading coefficient is 1.
2. Move the constant to the other side so the equation is in the form $x^2 + bx = c$.
3. Complete the square by taking half of the coefficient of *x*, squaring it, and adding to both sides of the equation.

4. Factor the perfect square trinomial.
5. Solve by using the square root property.

<u>Ex 4:</u> Solve by completing the square.

$a)\quad x^2+8x+9=0$

$b)\quad x^2-2=-6x$

$c)\quad x^2+x=6$

IV Solving by the Quadratic Formula

The Quadratic Formula is a formula that many of you will recognize. It can be used to solve any quadratic equation. How the formula is derived is found on page 143 of the text. (It is not important that you understand this derivation.)

Quadratic Formula

If a quadratic equation is in general form, $ax^2+bx+c=0$, then the solution(s) can be found by the formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$. It is read, '*x* equals the opposite of *b* plus or minus the square root of b^2-4ac all over twice *a*.'

Ex 5: Solve by using the quadratic formula.

$a)\quad 2x^2 - x - 15 = 0$

$b)\quad 5x + 3x^2 = -1$

$c)\quad 7x^2 = 2(x+1)$

$d)\quad x^2 - 15 = 0$

$e)\quad x^2 - 2x = -5$

$f)\quad x^2 - \frac{2}{3}x + \frac{2}{9} = 0$

V The Discriminant

The part of the quadratic formula under the radical sign $(b^2 - 4ac)$ is called the **discriminant.** (Some textbooks call it the determinant.) Note: The discriminant is without the radical sign.

1. If $b^2 - 4ac = 0$, then the formula would be $x = \frac{-b \pm \sqrt{0}}{2a} = -\frac{b}{2a}$. There will only be **one rational solution.**
2. If $b^2 - 4ac$ equals a positive perfect square number, then the formula would be $x = \frac{-b \pm \sqrt{k^2}}{2a} = \frac{-b \pm k}{2a}$, where k is a positive rational number. There will be **two rational solutions.**
3. If $b^2 - 4ac$ equals a positive non-perfect square number, then the formula would be $x = \frac{-b \pm \sqrt{n}}{2a}$, where n is a non-perfect square number. There will be **two irrational solutions** because the solutions include a radical.
4. If $b^2 - 4ac$ equals a negative number, then the formula will be $x = \frac{-b \pm \sqrt{-n}}{2a} = \frac{-b}{2a} \pm \frac{\sqrt{n}}{2a} i$, where n is a positive number. There will be **two complex solutions.**

Ex 6: Use the discriminant to determine the number and types of solutions.

$a)$ $x^2 + 6x = -9$

$b)$ $10x^2 + 29x = 21$

II Determining Which Method to Use

Always clear fractions and usually write a quadratic equation in general form with a positive leading coefficient.

There is a table on page 148 of the textbook that summarizes the most logical method to use when solving a quadratic equation. (However, they did not include completing the square method.) Here is my summary.

1. If the polynomial of $ax^2 + bx + c = 0$ can easily be factored, use factoring and the zero-product principle to solve.
2. If the equation is of the form $u^2 = d$, use the square root property to solve.

3. If the equation is of the form $x^2 + bx + c = 0$, you could use completing the square to solve.
4. The quadratic formula could always to used to solve, especially if it does not look easy to factor or the leading coefficient is not a 1.

<u>Ex 7:</u> Solve each equation by an appropriate method.

$a)\quad x - 2 = \frac{15}{x}$

$b)\quad 30x^2 - 10 = 13x$

$c)\quad (2x + 7)^2 = 36$

$d)\quad 3x^2 - 27 = 0$

$e)\quad x^2 - 4x = -17$

$f)\quad x(2x+5) = 2$