Summer MA 15200 Lesson 13 Section 1.6, Section 1.7 (part 1)

I Solving Polynomial Equations

Linear equation and quadratic equations of 1 variable are specific types of polynomial equations. Some polynomial equations of a higher degree can be solved by factoring.

1. Always look for a GCF first.
2. Factor using a difference of squares, trinomial methods, or grouping.

Ex 1: $6y^4 - 12y^2 = y^3$

Ex 2: $4x^4 + 9 - 37x^2 = 0$

Ex 3: $x^3 + 3x^2 - 36x - 108 = 0$

II Solving radical equations

Just as both sides of an equation may have the same number added or be multiplied by the same number, both sides of an equation may be raised to the same power. However, we must be careful.

Power Property for equations: When both sides of an equation are raised to the same power, the solutions that result ***may be*** solutions of the original equation. For example:

$x = 2$ solution is 2

$x^2 = 4$ solutions are -2 and 2

Raising both sides to the same power may result in an equation not equivalent to the original equation (different solution sets). These 'extra' solution or solutions (such as the -2 solution for the second equation) is/are known as **extraneous solution(s)** (solutions that don not satisfy the original equation). Therefore, whenever the power property for equations is used, **all possible solutions must be checked in the original equation.**

<u>Solving Radical Equations:</u>

1. Isolate the radical expression on one side of the equation or put one radical on each side.
2. Raise both sides of the equation to a power equal to the index.
3. Solve the result.
4. Remember to check all possible solutions in the original equation, since the power property for equations was used.

Hint: If a binomial is on the side opposite the radical side, FOIL must be used when squaring.

<u>Ex 4:</u> $\sqrt{x-2}+1=3$

<u>Ex 5:</u> $\sqrt{x-16}=\frac{3}{5}\sqrt{x}$

<u>Ex 6:</u> $\sqrt{5-x}=x+1$

<u>Ex 7:</u> $x-8x^{\frac{1}{2}}+12=0$

III Solving Basic Equations with Rational exponents

$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = x^1 = x$ This is the idea when solving with rational exponents. Raise both sides of the equation to the reciprocal power so that the variable expression is to the first power.

Solving equations of the form $x^{\frac{m}{n}} = k$

If m is even: $\left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = \pm k^{\frac{n}{m}} \rightarrow x = \pm k^{\frac{n}{m}}$

If m is odd: $\left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = k^{\frac{n}{m}} \rightarrow x = k^{\frac{n}{m}}$

Ex 8: Solve each. Remember to check.

$a)\quad x^{\frac{2}{3}} = 16$

$b)\quad (x+1)^{\frac{3}{2}} = 27$

IV Solving Absolute Value Equations

If $|x| = 2$, what value(s) could *x* equal?

You should see there could be 2 values for *x*, -2 or 2. Most absolute value equations will have two solutions, such as this equation. This is because an absolute value of a negative value or positive value would both be positive. Remember, absolute value means distance from zero and distance is always positive.

Exceptions will be equations such as $|x| = 0$ or $|x| = -5$. If an absolute value equals zero, there is only one value of *x* that will give zero, since only the absolute value of 0 is 0. An equation of the form absolute value equal a negative will never be true. This type of equation will always be 'no solution', inconsistent.

Solving Absolute Value Equations: $|x| = k$

1. If $k > 0$, then the equation becomes two linear equations $x = k$ and $x = -k$.
2. If $k = 0$, the equation becomes the linear equation $x = 0$.
3. If $k < 0$, there is no solution.

Ex 9: $|3x - 5| = 4$

Ex 10: $6 + 3|x + 5| = 15$ Hint: Isolate the absolute value.

Ex 11: $3 - |3x + 2| = -8$

Ex 12: $|3x| + 2 = 1$

V Formulas

Ex 13: The formula $M = 0.7\sqrt{x} + 12.5$ represents the average number of non-program minutes in an hour of prime-time cable *x* years after 1996. Project when there will be 16 minutes of non-program minutes of every hour of prime time cable TV, if this trend continues.

Ex 14: For each planet in our solar system, its year is the time it takes the planet to revolve once around the sun. The formula $E = 0.2x^{\frac{3}{2}}$ models the number of Earth days in a planet's year, where *x* is the distance of the planet from the sun, in millions of kilometers. Assume a planet has an orbit equal to 200 earth days. Approximate the number of kilometers that planet is from the sun.

VI Representing an Inequality

There are 3 ways to represent an inequality. (1) Using the inequality symbol (sometime within set-builder notation), (2) using interval notation, and (3) using a number line graph.

The following table illustrates all three ways. Notice that interval notation looks like an ordered pair, sometimes with brackets. When writing the ordered pair, always write the lesser value to the left of the greater value. A parenthesis next to a number illustrates that *x* gets very, very close to that number, but never equals the number. A bracket next to a number means it can equal that number. With ∞ or $-\infty$ a parenthesis is always used, since there is not an exact number equal to ∞ or $-\infty$.

INEQUALITIES

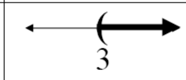
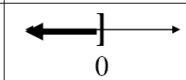
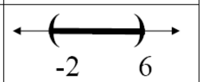
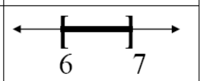
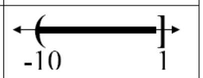
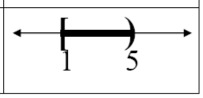
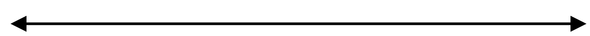
Inequality	Interval Notation	Graph
$x > 3$	$(3, \infty)$	
$x \le 0$	$(-\infty, 0]$	
$-2 < x < 6$	$(-2, 6)$	
$6 \le x \le 7$	$[6, 7]$	
$-10 < x \le 1$	$(-10, 1]$	
$5 > x \ge 1$	$[1, 5)$	

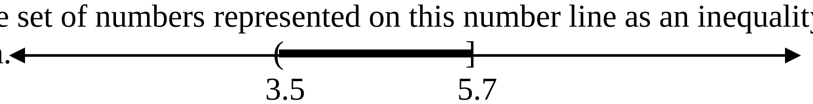
Table 1.4 on page 174 of the textbook also illustrates the 3 ways to represent an inequality where $a < b$.

<u>Ex 15:</u> Write this inequality in interval notation and graph on a number line.

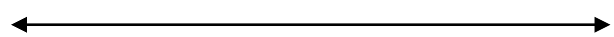
$\{x \mid x > 1\}$



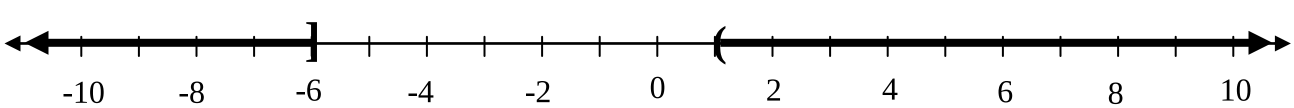
<u>Ex 16:</u> Write the set of numbers represented on this number line as an inequality and in interval notation.



<u>Ex 17:</u> Write the following as an inequality and graph on a number line. $(-\infty, 5]$



Examine the number line below.



In set-builder notation, it would be represented as $\{x \mid x \le -6 \text{ or } x > 1\}$ and in interval notation, it would be represented as $(-\infty, -6] \cup (1, \infty)$.

VII Solving a Linear Inequality in One Variable

Solving linear inequalities is similar to solving linear equation, with one exception. Examine the following.

$5 < 10$

Add -6 to both sides: $-1 < 4$ True

Multiply both sides by 2: $10 < 20$ True

Divide both by 5: $1 < 2$ True

However, try multiplying by -2: $-10 < -20$ False

Divide by -5: $-1 < -2$ False.

This leads to the following properties of Inequalities

1) **The Addition Property of Inequality**

If $a < b$, then $a + \boldsymbol{c} < b + \boldsymbol{c}$

$$a - \boldsymbol{c} < b - \boldsymbol{c}$$

2) **The Positive Multiplication Property of Inequality**

If $a < b$ and c is positive, then $a\boldsymbol{c} < b\boldsymbol{c}$

$$\frac{a}{\boldsymbol{c}} < \frac{b}{\boldsymbol{c}}$$

3) **The Negative Multiplication Property of Inequality**

If $a < b$ and c is negative, then $a\boldsymbol{c} > b\boldsymbol{c}$

$$\frac{a}{\boldsymbol{c}} > \frac{b}{\boldsymbol{c}}$$

Ex 18: Solve each inequality. Write the solutions with the inequality symbol, in interval notation, and graph the solutions on a number line.

$a)\quad 3(x+2) \le -4(5-x)$

$b)\quad \dfrac{6(x-4)}{5} > \dfrac{3(x+2)}{4}$

$c)\quad \dfrac{5}{9}(a+3) - a \ge \dfrac{4}{3}(a-3) - 1$

It the variables ‘drop out’ of a linear inequality, the solution is either all real numbers (except for any that may not be in the domain) or there is no solution.

$3(x+2) < 3x+7$

$3x+6 < 3x+7$

$6 < 7$

The result above is always true, 6 is less than 7. The solution is $\{x \mid x \text{ is a real number}\}$ or $\mathbb{R}$ or $(-\infty, \infty)$.

$3(x+2) < 3x+2$

$3x+6 < 3x+2$

$6 < 2$

Six is never less than 2. The result is false. The solution is $\varnothing$ or no solution.

III Solving Compound Inequalities

When solving an inequality such as $-12 < 3x + 3 < 2$, the goal is to **isolate the *x* in the middle.** Such an inequality is called a compound inequality and means the same as $-12 < 3x + 3$ and $3x + 3 < 2$. The solution will be the numbers that, when substituted in $3x + 3$, yields between -12 and 2.

$-4 < 2x - 1 < 5$

Begin by adding 1 to the left, middle, and right.

Example: $-3 < 2x < 6$

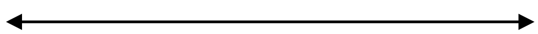
Divide the left, middle, and right by 2.

$-\frac{3}{2} < x < 3$

Any number between -1 ½ and 3 makes the inequality statement true.

<u>Ex 19:</u> Solve each compound inequality. Write the solutions using the inequality symbols, in interval notation, and graph the solutions on a number line.

$a)$ $1 < 3x - 2 < 12$



$b)$ $-2 < 6 - 3x < 3$