Summer MA 15200 Lesson 14Section 1.7 (part 2) and parts of sections 1.1 & 2.8

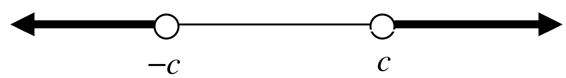
I Solving Absolute Value Inequalities

Absolute Value Inequalities: $|u| < c$ or $|u| \le c$, if $c \ge 0$

The inequality $|u| < c$ indicates all values less than c units from the origin. Therefore $|u| < c$ is equivalent to the compound inequality $-c < u < c$. There is a similar statement for $|u| \le c$.

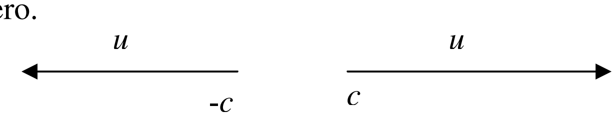
Absolute Value Inequalities: $|u| > c$, $|u| \ge c$, if $c > 0$

The inequality $|u| > c$ indicates all values more than c units from the origin. Therefore $|u| > c$ is equivalent to the inequality statement $u < -c$ or $u > c$. There is a similar statement for $|u| \ge c$.

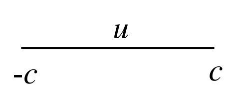


To help you keep the two cases straight in your head, I recommend thinking of a number line.

If the absolute value is greater than a positive number c, it is greater than that many units away from zero.



If the absolute value is less than a positive number c, it is within that many units of zero.



Ex 1: Solve each. Write solutions using interval notation and graph the solutions on a number line.

Hint: Always isolate the absolute value before writing an inequality without the absolute value.

$a)\quad |x+4| < 6$

$b)\quad |3x+5|+1 \le 9$

$c)\quad \left|\frac{5x+2}{3}\right| < 1$

Ex 2: Solve each inequality. Write the solutions using interval notation and graph the solutions on a number line.

$a)\quad |2-x| > 5$

$b)\quad \left|\frac{x}{2}+3\right| \ge 7$

$c)\quad |3x-4|+2 > 7$

II Applied Problems

Ex 3: Mary wants to spend **less than** \$600 for a DVD recorder and some DVDs. If the recorder of her choice costs \$425 and DVDs cost \$7.50 each, how many DVDs could Mary buy?

Ex 4: The percentage, *P*, of US voters who used punch cards or lever machines in national elections can be modeled by the formula $P = -2.5x + 63.1$ where x is the number of years after 1994. In which years will fewer than 35.7% of US voters use punch cards of lever machines?

Ex 5: A college provides its employees with a choice of two medical plans shown in the following table.

Plan 1:	\$100 deductible payment	30% of the remaining payments
Plan 2:	\$200 deductible payment	20% of the remaining payments

For what size hospital bills is plan 2 better for the employee than plan 1? (Assume the bill is over \$200.)

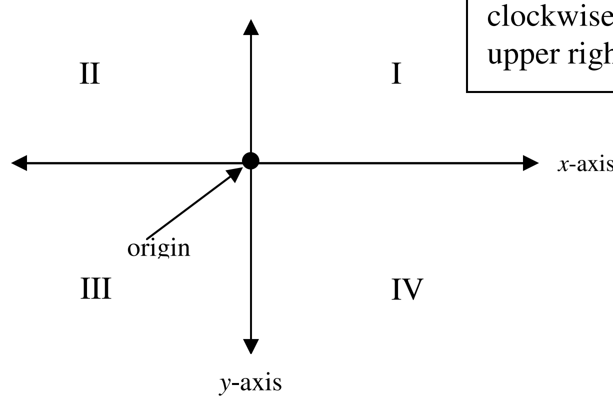
Ex 6: The room temperature in a public courthouse during a year satisfies the inequality $|T-71|<3$ where T is in degrees F. Express the range of temperatures without the absolute value symbol.

There are **4 quadrants**, represented by Roman Numerals counter-clockwise from the upper right.

III Rectangular Coordinate System

Rectangular Coordinate System:

Every point on a rectangular coordinate system is represented by an **ordered pair** , (*x*, *y*). *x* and *y* are called the **coordinates** of the point.



Ex 7:

a) In which quadrant(s) do the coordinates of a point have the same sign?

b) In which quadrant would the point (-2, 3) be found?

c) The point (12, 0) is found on which axis?

d) What point would be 5 right and 6 down from the origin?

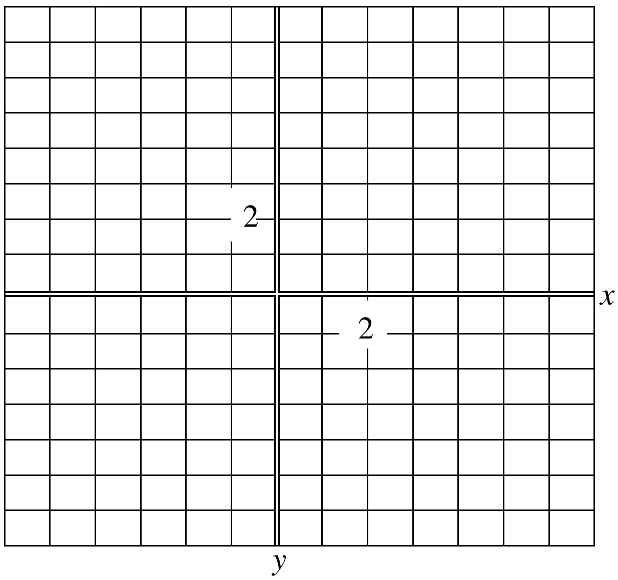
IV Graphs of Equations

An equation in 2 variables can be represented on a rectangular coordinate system by plotting points (ordered pairs) that satisfies the equation. The complete graph contains all ordered pairs whose coordinates satisfy the equation.

1. Make a table by selecting a value for *x* and solving for *y* (or vice-versa).
2. Plot enough points to be able to sketch a smooth curve or line to represent the graph.

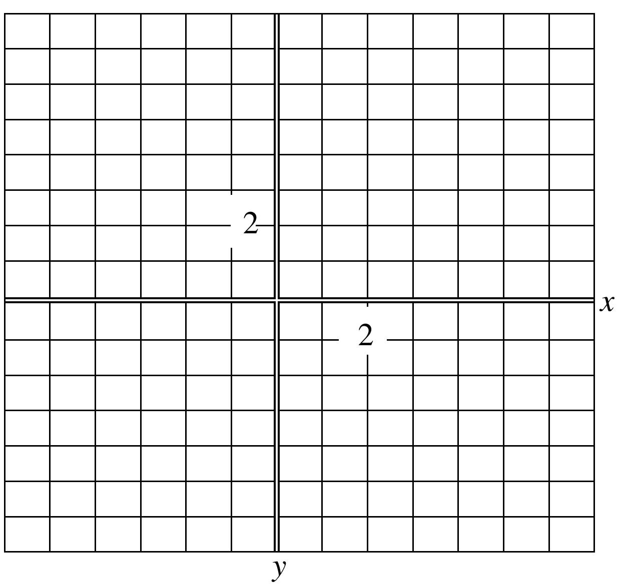
Ex 8: Graph the following equations. Use the values of $-3, -2, -1, 0, 1, 2$ and 3 for x.

a) $y = 3 - x^2$



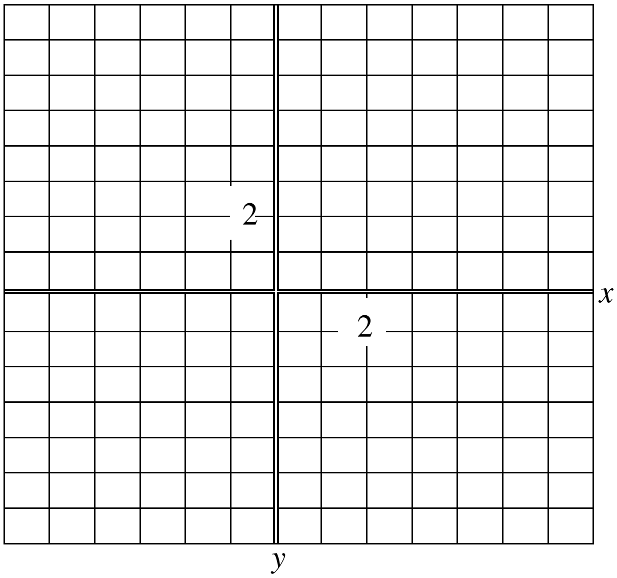
x	y

b) $y = \left|\frac{1}{2}x\right|$



x	y

$c)\quad y = -\frac{1}{2}x^3 + 1$



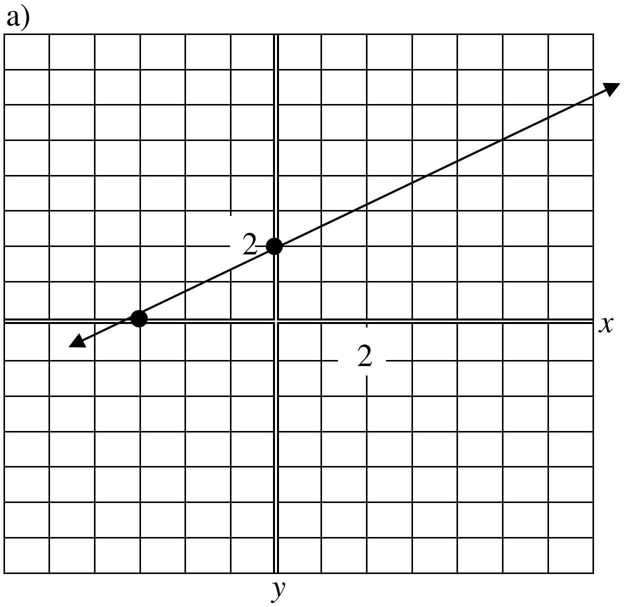
x	y

V Intercepts

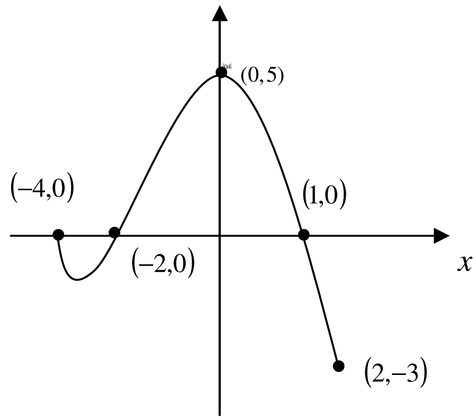
An ***x*-intercept of a graph** is an *x*-coordinate of a point where the graph intersects the *x*-axis. The ***y*-intercept of a graph** is the *y*-coordinate of a point where the graph intersects the *y*-axis. Yes, our textbook uses single numbers to describe intersepts. Some textbooks use the complete ordered pair to define the intercepts. (See the study tip at the bottom of page 94 of the textbook.) Since an intercept always lies on an axis, the coordinate other than the one given as the *x*-intercept or *y*-intercept is zero. Zero is your friend!

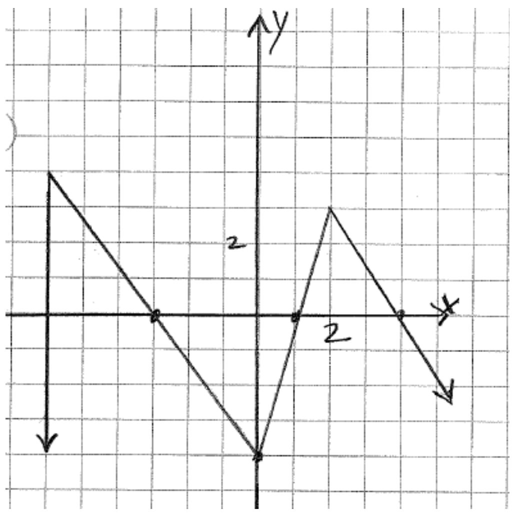
<u>Ex 9:</u> Use the following graphs to identify the intercepts.

a)

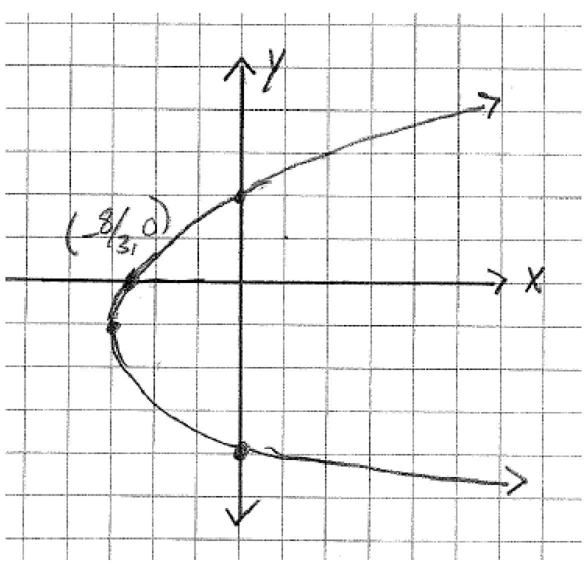


b)





c)



d)

VI Applied Problems

On page 96 of your textbook is a graph showing the probability of divorce by the wife's age at the time of marriage. The two mathematical models that approximate the data displayed in the graphs are $d = 4n + 5$ (where d = the percentage of marriages ending in divorce for n years after the marriage for a wife under 18 at time of marriage) and $d = 2.3n + 1.5$ (for a wife over 25 at time of marriage).

Ex 10: a) Use the graph to approximate the percentage of marriages ending in divorce after 5 years of marriage if the wife was under 18.

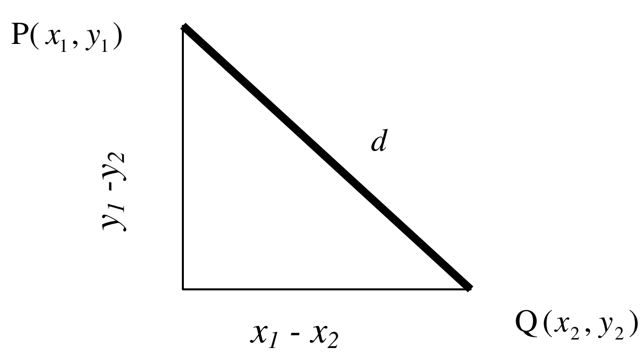
b) Use the correct model (formula) to approximate the percentage of marriages ending in divorce after 5 years of marriage if the wife was under 18.

c) Does the value given by the mathematical model underestimate or overestimate the actual percentage of marriages ending in divorce after 5 years for a bride under 18 shown in the graph? By how much?

VII Distance Between 2 Points on a Coordinate System

Notation for points: Points are labeled using capital letters. Sometimes a point may be written as follows: $P(x_1, y_1)$. This is read 'point P with coordinates of x sub 1 and y sub 1.

To find the distance between two points, the Pythagorean Theorem could be used.



Distance Formula:

$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

The second line is known as the **distance formula** between two points.

Ex 11: Find the exact distance (in simplified form) between the 2 given points.

$a)$ $P(0,5)$, Q(6,-3)

b) $P(3,-3)$, $Q(-5,5)$

Ex 12: Approximate the distance between the 2 given points to the nearest hundredth.
$(2.6,-3.1)$, $(-8.5, 2.1)$

Midpoint Formula:

The **midpoint** of two point P and Q is the point midway between P and Q. Its coordinates are the average of the x coordinates of P and Q and the average of the y coordinates of P and Q.

The midpoint of $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Study Tip page 295: The midpoint requires sum of the coordinates. The distance formula requires difference of the coordinates.

Ex 13: Find the midpoint of each pair of points.

$a)$ $P(0,5)$, Q(6, −3)

b) $P(4,-3)$, $Q\left(-5, \frac{3}{2}\right)$

(optional)

Ex 14: Determine is a triangle with the following vertices is an **equilateral triangle**.

$A(13,-2),\ B(9,-8),\ C(5,-2)$