Summer MA 15200 Lesson 15 Section 2.1, Section 2.2 (part 1)

A relation is any set of ordered pairs. The set of all first components of the ordered pairs is called the **domain**. The set of all second components is called the **range.**

Relations can be represented by tables, sets, equations of two variables, or graphs.

Name	% of all Names
Smith	1.006%
Johnson	0.810%
Williams	0.699%
Brown	0.621%
Jones	0.621%

The table at the left would represent a relation where the ordered pairs are of the form (name, %). An example would be (Williams, 0.699%). The domain would be {Smith, Johnson, Williams, Brown, Jones} and the range is {1.006%, 0.810%, 0.699%, 0.621%}.

Ex 1: Find the domain and range of each relation.

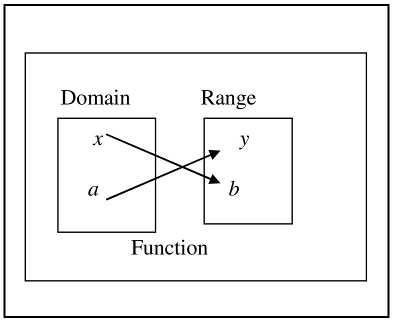
a) $\{(2,-3),(3,-4),(4,-5),(5,-5),(2,-6)\}$

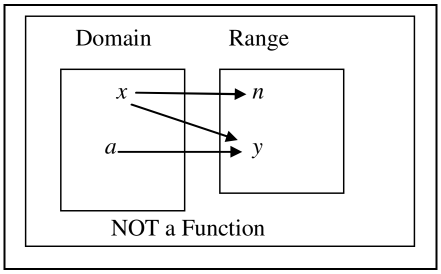
A relation in which each member of the domain corresponds to exactly one member of the range is a **function**. The table above that pairs a last name with a percent of all names is a function because each last name is paired to exactly one percent. Another way to identify a function is the following, it is a **relation in which no two ordered pairs have the same first component and different second components.**

Definition of a Function
A **function** is a correspondence from a first set, called the **domain**, to a second set, called the **range**, such that each element in the domain corresponds to ***exactly one*** element in the range.

If the table was changed as below, the relation is not a function. The percent 0.621% would be paired with both Brown and Jones.

% of all Names	Name
1.006%	Smith
0.810%	Johnson
0.699%	Williams
0.621%	Brown
0.621%	Jones





I Determining Whether a Relation is a Function

Ex 2: Determine if each relation is a function.

a)

x	y
0	1
1	0
-1	0
2	-3
-3	-8
-2	-3
4	-15

b) $\{(2,-3),(3,-4),(4,-5),(5,-5),(2,-6)\}$

II Determining Whether an Equation Represents a Function

Many functions are written as equations of two variables. For example, $R=-0.6x+94$, where *R* represents the average number of meals per person Americans ordered from restaurants and *x* represents the number of years after 1984. The *x* is called the **independent variable**, because any number of years after 1984 can be selected. The *y* is called the **dependent variable,** because its value depends upon the value of *x*.

Not all equations represent functions. If an equation is solved for *y* and more than one value of *y* can be obtained for a given *x*, then the equation does not define a function.

Ex 3: Determine if each equation is a function or not. Find the domain in interval notation.

a) $y=x^2+2$

c) $x=y^2+2$

b) $y=\sqrt{x-3}$

d) $xy-y=-1$

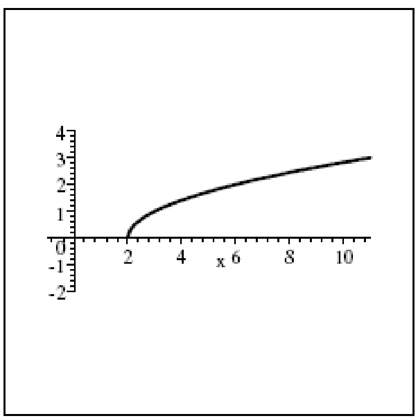
e) $y = \dfrac{2x}{x-5}$

III Determining if a Graph Represents a Function

Graphs of Functions: A function can be graphed by determining the set of all ordered pairs (points) where *x* is in the domain and *y* is in the range. Because each *x* can only be paired to one *y*, the **vertical line test** can be used to determine if a graph represents a function. If every possible vertical line would intersect the graph only once, then the graph represents a function.

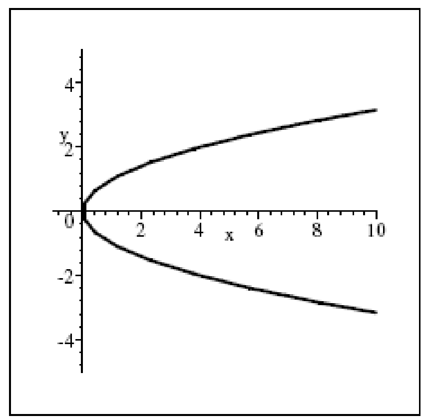
<u>Ex 2:</u> Determine which graphs are functions.

a)



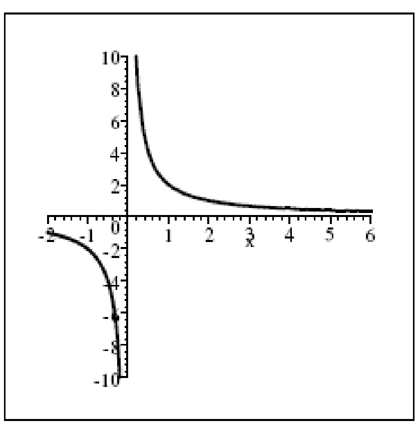
$y = \sqrt{x-2}$

b)



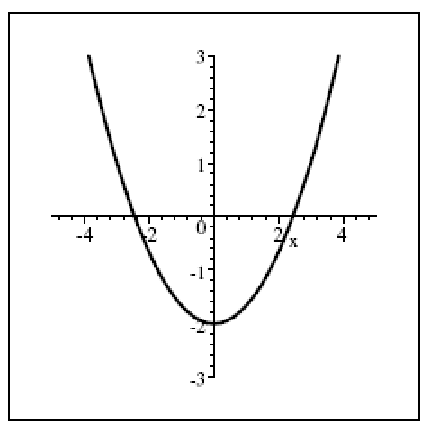
$y^2 = x$

c)



$y = \frac{2}{x}$

d)



$y = \frac{1}{3}x^2 - 2$

The Vertical Line Test for Functions
If any vertical line intersects a graph in more than one point, the graph does not define *y* as a function of *x*.

IV Function Notation

Functions are often named using letters such as *f*, *g*, *h, F, G, or p*. The input of the function is *x*. The output of the function is *y*, which can be represented as *f*(*x*), read *f* of *x* or function of *x*. This notation means the **value of the function at the number *x*** and is known as **function notation.**

The notation *f*(2) means replace a 2 for the value of *x* (or the independent variable) in the function. Some substitutions for *x* do not have to be numbers. *f(x-4)* says to find the **function value** when the original *x* is replaced with *x* – 4. Remember *f*(*x*) means the same thing as *y*. The function value is the *y* or *f(x)* value.

Ex 3: If $f(x) = 3x^2 - 2x$ and $g(x) = 4x + 2$, find the following.

$a)$ $f(-1) =$

$b)$ $g(-4) =$

$c)$ $f(a) =$

$d)$ $g(2k) =$

$e)$ $g(m-2) =$

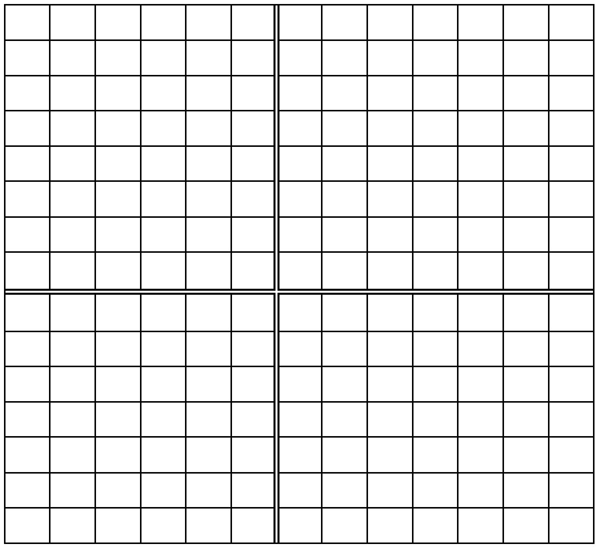
$f)$ $f(n^2+1) =$

$g)$ $g(x+5) =$

V Graphs of Functions

The graph of a function is the graph of its ordered pairs. We have already graphed some equations, but will graph one function here.

Ex 4: Graph $g(x) = 2x - 3$

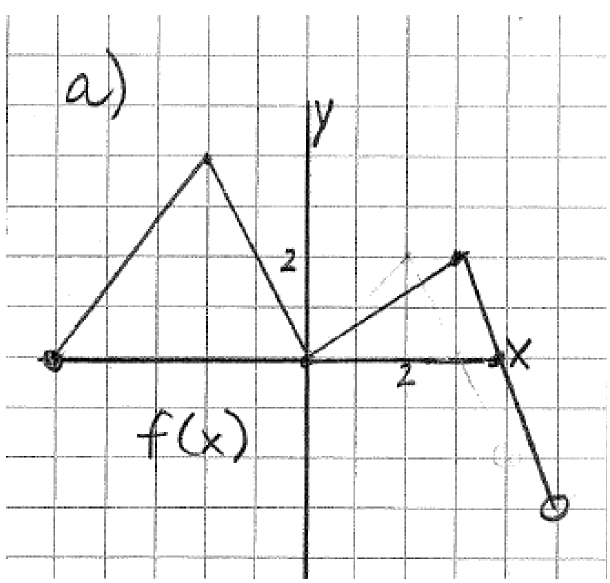


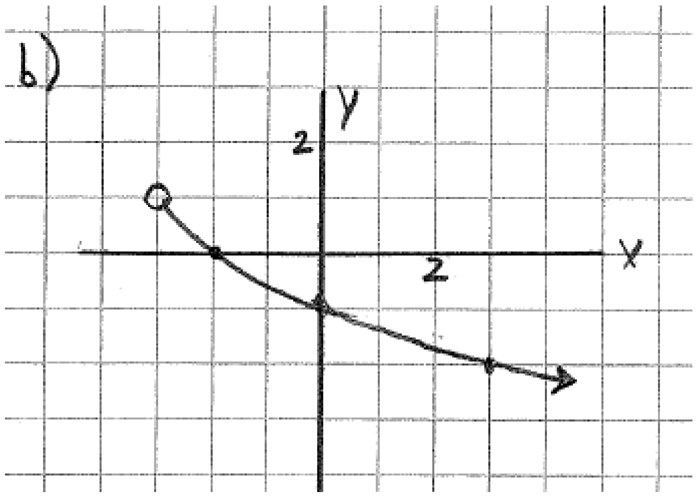
The graph above is a line. Any function of the form $f(x) = mx + b$ has a straight line graph and is called a **linear function.**

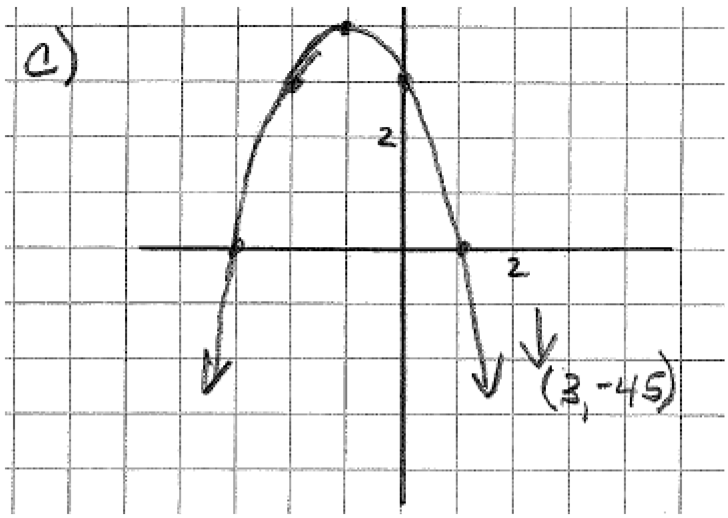
VI Analyzing the Graph of a Function

Ex 5: For each graph shown, find the following.

a) The Domain in set-builder notation and interval notation.
b) The Range in set-builder notation and interval notation.
c) $f(3)$
d) For what value of x is $f(x) = 4$?
e) Any x-intercepts
f) Any y-intercept

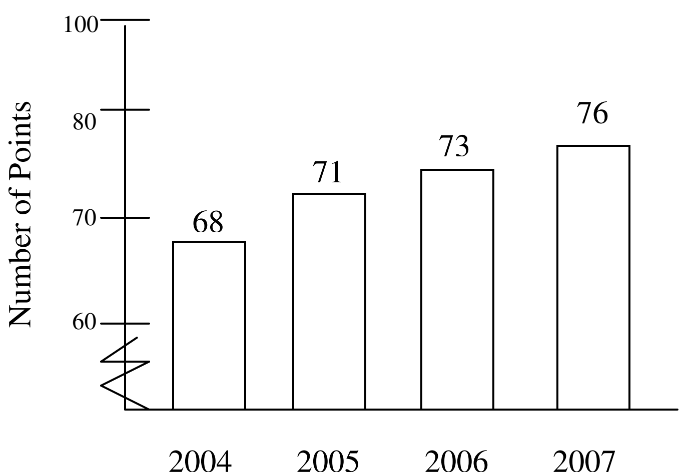






VII Applied Function Problems

The bar graph below shows the average final exam score for 4 consecutive years.



The functions $f(x) = 3x + 56$ and $g(x) = \frac{2}{5}x^2 + 56$ model the average exam score where x is the number of years after 2000.

Ex 6: a) According to the graph, what was the average exam score in 2007?

b) Using the function *f* model, what was the average exam score in 2007?

c) Does function *f* underestimate or overestimate the actual exam score according to the graph? By how much?

d) Using the function *g* model, what was the average exam score in 2007?

e) Does the function *g* overestimate or underestimate the actual average given in the graph? By how much?

VIII Increasing, Decreasing, or Constant Functions

A function is **increasing** if in an open interval, whenever $x_1 < x_2$, then $f(x_1) < f(x_2)$. The function values (or points on graph) are always **rising.**

A function is **decreasing** if in an open interval, whenever $x_1 < x_2$, then $f(x_1) > f(x_2)$. The function values (or points on graph) are always **falling**.

A function is **constant** if in an open interval, $f(x_1) = f(x_2)$ for any x_1 and x_2 in the interval. The function values are equal and the graph is **flat.**

When describing intervals where function is increasing or decreasing, use from **one *x* to the next *x*.**

When describing the interval where a function is increasing, decreasing, or constant; the intervals are open (no brackets) and you use the ***x*-coordinates**.

IX Relative Maxima and Relative Minima

A function has a **relative maximum** value $f(a)$ if there is an open interval containing a such that $f(a) > f(x)$ for all $x \neq a$ in the open interval. In other words, the function value $f(a)$ is larger than all other function values in that interval. Graphically, this is the function value of the **highest point** of the interval of the graph of the function.

An endpoint can never be a relative maximum or relative minimum.

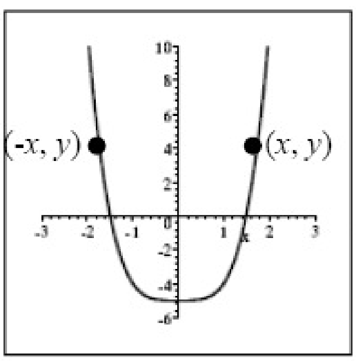
A function has a **relative minimum** value $f(b)$ if there is an open interval containing b such that $f(b) < f(x)$ for all $x \neq b$ in the open interval. In other words, the function value $f(b)$ is smaller than all other function values in that interval. Graphically, this is the function value of the **lowest point** in the interval of the graph of the function.

We say there is a relative maximum or relative minimum value $f(a)$ or $f(b)$ at a certain value of x. The y value of a point is the relative maximum or relative minimum.

X Even or Odd Functions

A function f is an **even function** if $f(-x) = f(x)$ for all x in the domain of f. The ordered pairs $(x, f(x))$ and $(-x, f(x))$ will both be in the function. Graphically, **even functions have symmetry about the *y*-axis or symmetry with respect to the *y*-axis.**

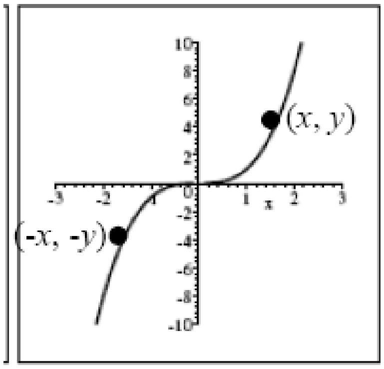
Here is an example of symmetry with respect to the y-axis.



symmetry about the y-axis

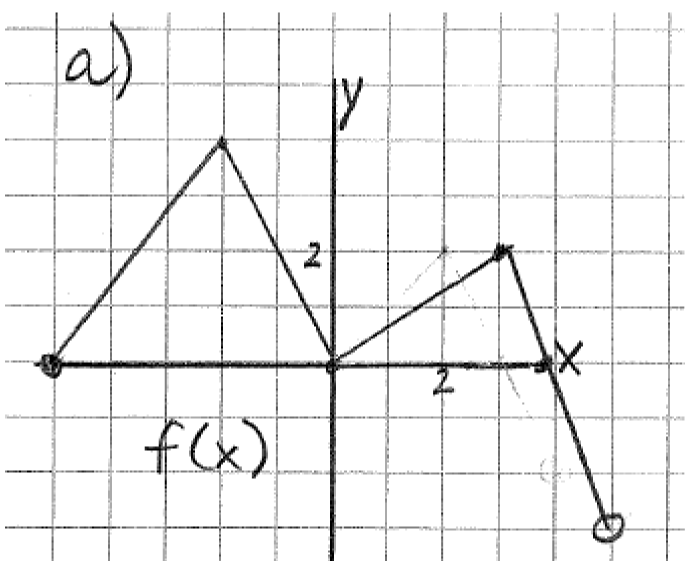
The function f is an **odd function** if $f(-x) = -f(x)$ for all x in the domain of f. The ordered pairs $(x, f(x))$ and $(-x, -f(x))$ will both be in the function. Graphically, **odd functions have symmetry about the origin or symmetry with respect to the origin.** This means the origin would be the midpoint of the line segment connecting the two points.

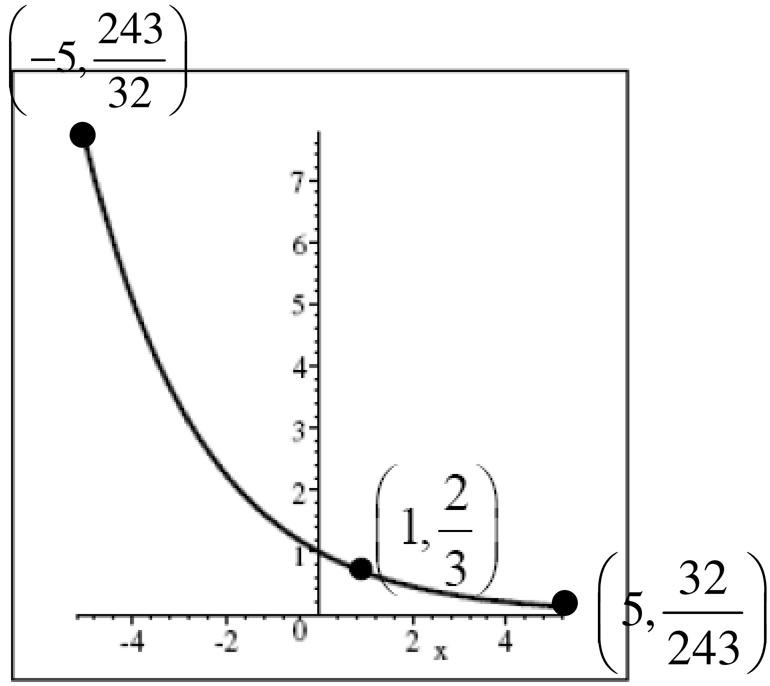
Here is an example of symmetry with respect to the origin.



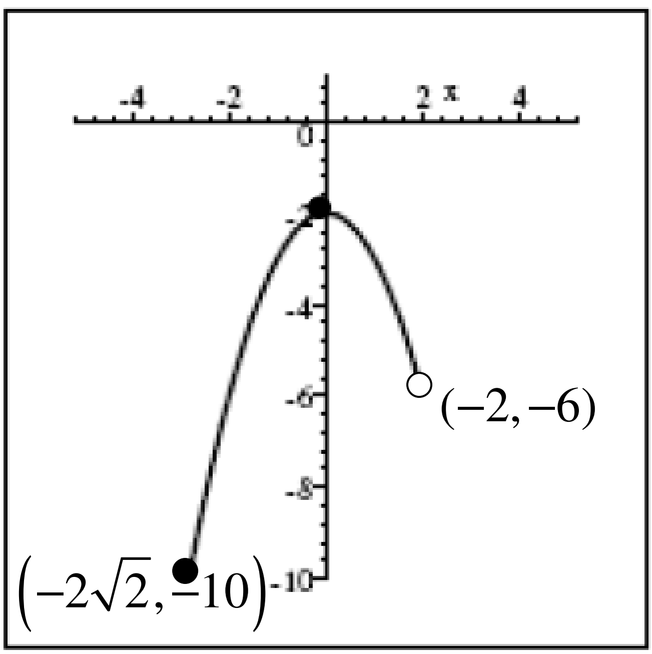
Ex 7: For each graph determine the following.

a) The intervals in which the function is increasing, if any.
b) The intervals in which the function is decreasing, if any.
c) The intervals in which the function is constant, if any.
d) The relative maximum(s), if any.
e) The relative minimum(s), if any.
f) Describe if the function as even, odd, or neither (check symmetry).
g) The zeros of the function (zeros are the x-intercepts)

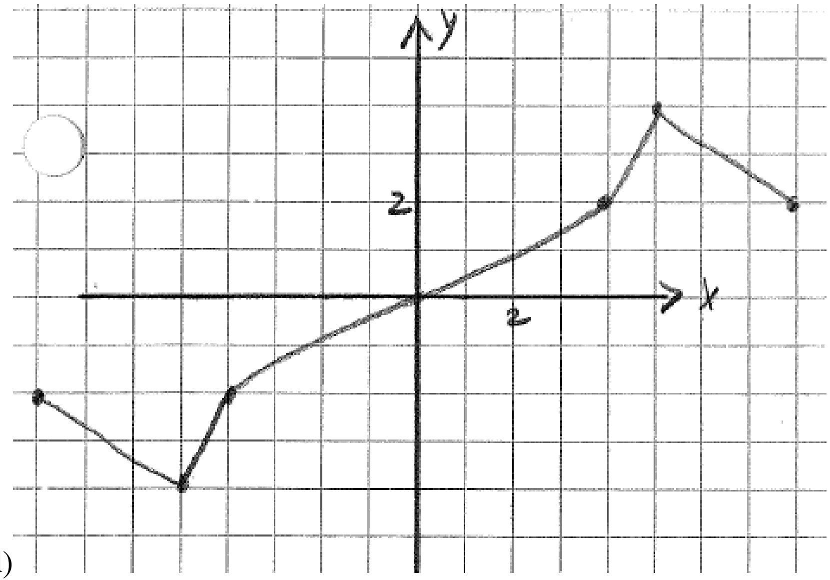



$\left(-5, \frac{243}{32}\right)$
$\left(1, \frac{2}{3}\right)$
$\left(5, \frac{32}{243}\right)$
7
6
5
4
3
2
1
-4
-2
0
2
x
4

b)


-4
-2
2
x
4
0
-4
-6
-8
-10
$(-2, -6)$
$\left(-2\sqrt{2}, -10\right)$

c)

d) 

Ex 8: Describe each function as even, odd, or neither.

$a)\quad f(x) = x^5 - x^3$

$b)\quad g(x) = 4x^4 - 12$

$c)\quad h(x) = 2x^3 - 3x + 2$

$d)\quad f(x) = |x| + 5$

$e)\quad g(x) = 3x^3\sqrt{x^2+1}$