MA 15200 Lesson 16 Sections 2.3 and 2.4

I Piecewise Functions & Evaluating such Functions

A cab driver charges \$4 a ride for a ride less than 5 miles. He charges \$4 plus \$0.50 a mile for a ride greater than or equal to 5 miles. This situation could be described by the function $f(m)=\begin{cases} 4 & \text{if } 0<m<5 \\ 4+0.5m & \text{if } m\geq 5 \end{cases}$, where m represents the number of miles. Such a function is called a **piecewise function.** A piecewise function is made up of part of two or more functions, each with its own domain.

Ex 1: For each piecewise function, find $f(-4),\ f(0),\ and\ f(2)$, if defined.

a) $f(x)=\begin{cases} 2 & \text{if } x\leq 0 \\ x+2 & \text{if } x>0 \end{cases}$

b) $f(x)=\begin{cases} 2x+1 \text{ if } x<-4 \\ 3x-9 \text{ if } -4\leq x<0 \\ 5x+3 \text{ if } x\geq 0 \end{cases}$

Ex 2: A cellular phone plan has the function below to describe the total monthly cost where t represents the number of calling minutes.

$C(t)=\begin{cases} 25 & \text{if } 0\leq x\leq 120 \\ 25+0.30(t-120) & \text{if } x>120 \end{cases}$

Find the following values and interpret them.

$a)$ $C(100)$

$b)$ $C(120)$

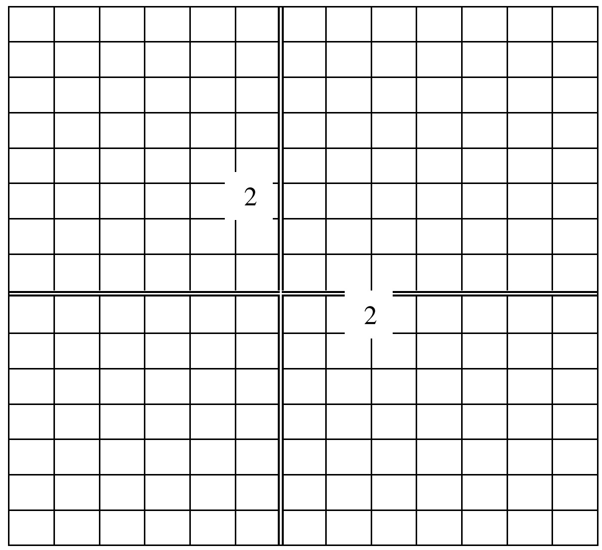
$c)$ $C(140)$

II Graphing Piecewise Functions

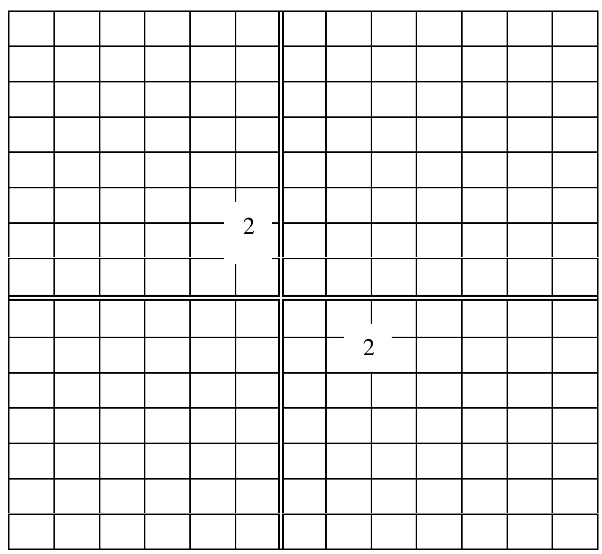
To graph a piecewise function, use a partial table of coordinates to create each piece. For 'endpoints', use the appropriate open or closed circle. An **open circle** is used when the *x* value cannot equal the given value, only approach it. A **closed circle** is used when the *x* value can equal the given value.

<u>Ex 3:</u> Graph each piecewise function.

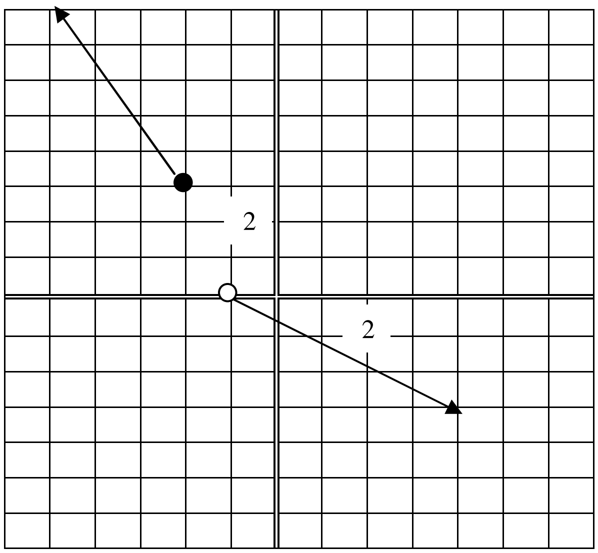
a) $f(x) = \begin{cases} 5 & \text{if } x < 1 \\ x+1 & \text{if } x \geq 1 \end{cases}$



b) $g(x) = \begin{cases} x^2 & \text{if } x \leq -1 \\ -x & \text{if } -1 < x < 2 \\ 2x & \text{if } x \geq 2 \end{cases}$



Ex 4: Describe the domain and range of the piecewise function below.



III Applied Problems

Ex 5: Use the graph before problem 83 on page 227 of the textbook.

a) What is the **range** for the 'women' graph?

b) At **what age** does the percent of body fat reach a **maximum** for men?

c) What is the difference of the percent of body fat for men and women at age 35?

Ex 6: In a certain city, there is a local income tax that is described by the table below.

If your taxable income is over..	But not over...	The tax you owe is..	Of the amount over..
$0	$10,000	1%	$0
$10,000	$25,000	$100 + 2%	$10,000
$25,000	-	$300 + 3%	$25,000

Write a piecewise function to describe the tax, where x is income.

$$T(x) = \begin{cases} 0.01x & if \quad x \le 10,000 \\ 100 + 0.02(x - \square) & if \quad 10,000 < x \le 25,000 \\ 300 + \square\,(x - \square) & if \quad x > 25,000 \end{cases}$$

IV Slope of a Line

A measure of the 'steepness' of a line is called the **slope** of the line. Slope compares a vertical change (called the **rise**) to the horizontal change (called the **run**) when moving from one point to another point along a line.

Slope is a ratio of vertical change to horizontal change.

If a non-vertical line contains points (x_1, y_1) and (x_2, y_2), the slope of the line is the ratio described by

$$m = \frac{rise}{run} = \frac{\text{change in y}}{\text{change in x}} = \frac{y_2 - y_1}{x_2 - x_1}.$$

*Note: Always be consistent in the order of the coordinates.

> When given two points, it does not matter which one is called point 1 and which point 2.
> $$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

There are 3 ways to find slope.

1. Using the slope formula (above)
2. Counting rise over run (when shown a graph)
3. Finding the equation in slope-intercept form (later in lesson)

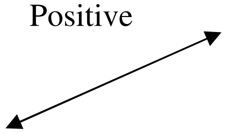
If a line is horizontal, the numerator in the slope formula will be 0 (the y coordinates of all points of a horizontal line are the same). The slope of a horizontal line is 0.

If a line is vertical, the denominator in the slope formula will be 0 (the x coordinates of all points of a vertical line are the same). A number with a zero denominator is not defined or undefined. The slope of a vertical line is undefined.

> Never say 'no slope' to define the slope of a vertical line. No slope could be interpreted as 0 or undefined.

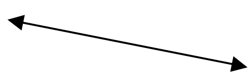
There are 4 types of slopes.

Positive



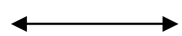
line rises left to right

Negative



line falls left to right

Zero



horizontal line

Undefined

vertical line

Ex 7: Find the slope of a line containing each pair of points.

Describe if the line rises from left to right, falls from left to right, is horizontal, or is vertical.

$a)\quad P(2,-3),\ \ Q(-6,-12)$

$b)\quad P(-4,2),\ \ Q(5,3)$

$c)\quad P(-1,0),\ \ Q(2,1)$

$d)\quad P(-4,10),\ Q(-4,-8)$

$e)\quad P(6,-2),\ Q(9,-2)$

VII Slope-Intercept Form

If a line has a slope *m* and a *y*-intercept of *b* (point (0, *b*)), then the equation of the line can be written as $y = mx + b$. This is known as **slope-intercept form of the equation of a non-vertical line.** This can also be written as $f(x) = mx + b$ and is a **linear function.**

Ex 8: Find the slope of each line given its equation.

$a)\quad y = -4x + 2$

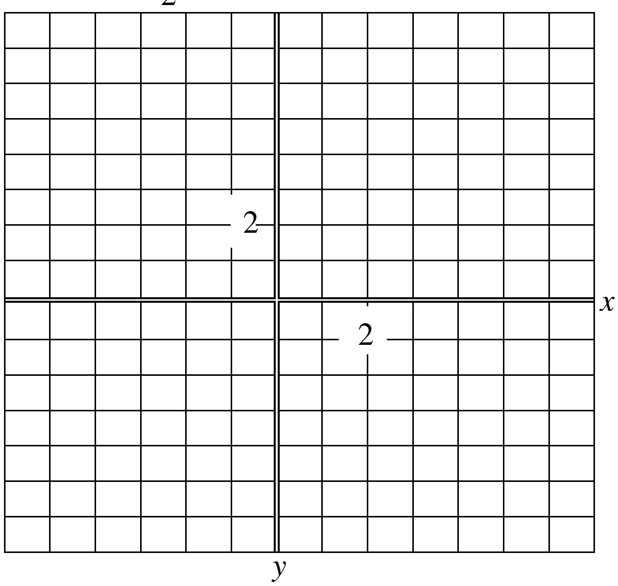
$b)\quad 7x - 8y = 12$

VIII Graphing a Line using slope and y-intercept

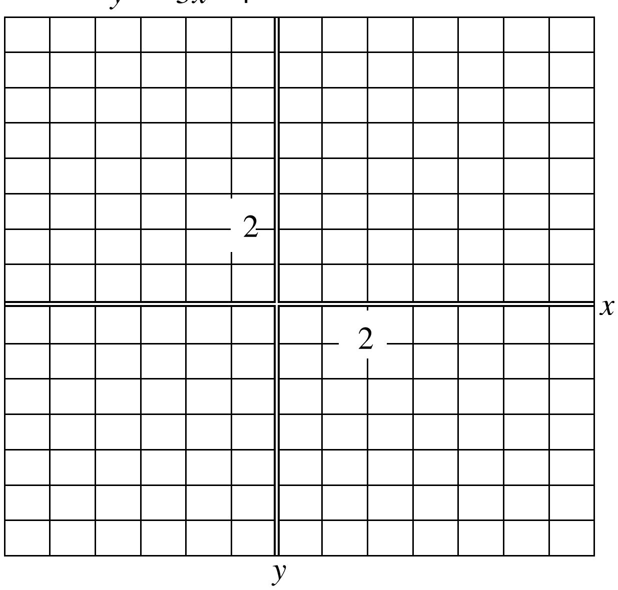
1. Plot the y-intercept on the *y*-axis (0, *b*)
2. Obtain a second point using the slope *m*. Write *m* as a fraction and use rise over run, starting at the *y*-intercept. (Note: If the slope is negative, let the rise be negative and the run positive. Move down and then right. If you let the run be negative and rise positive, move up and then left.)
3. Connect the two points to draw the line. Put arrows at each end to indicate the line continues indefinitely in both directions.

Ex 9: Graph each line.

$y=\frac{1}{2}x+2$



$y=-3x-4$



IX Equations and Graphs of Horizontal or Vertical Lines

It a line is horizontal, the slope-intercept form is written $y = 0x + b$ or $y = b$. A vertical line cannot be written in slope-intercept form because there is no possible number for *m*. However, a vertical line would have points all with the same *x*-coordinate. So a vertical line can be written as $x = a$, where *a* is the *x*-intercept.

If *a* and *b* are real numbers, then

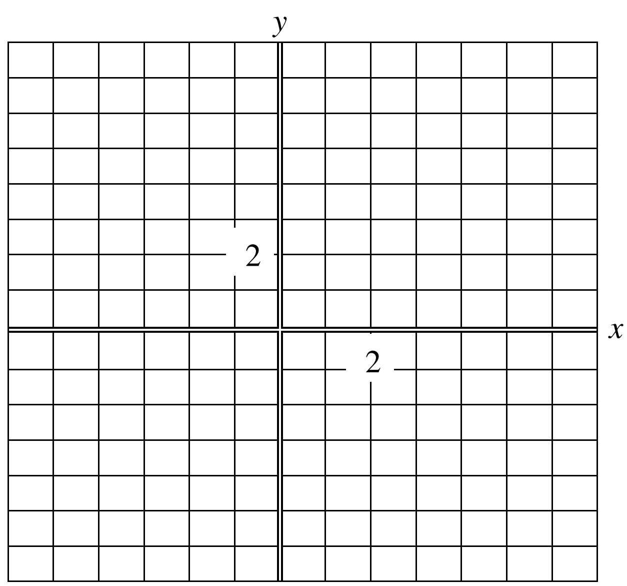
- The graph of the equation $x = a$ is a vertical line with an *x*-intercept of *a*.
- The graph of the equation $y = b$ is a horizontal line with a *y*-intercept of *b*.

Note: If the equation has only an *x* or only a *y*, solve for that variable. Then you will know where the intercept is and be able to graph the line.

Ex 10: Graph each line.

$a)\quad x = -3$

$b)\quad y = 4$



X Slope as a Average Rate of Change

Slope is a ratio, described as a change in *y* compared to a change in *x*. It describes how quickly *y* is changing with respect to *x*. For data that models a line within a certain interval, slope describes what is known as **average rate of change.** For example suppose in the year 2000 a town had 10,000 persons and in the year 2005, the town had 15,000 persons. This data could be written as the ordered pairs (2000, 10000) and (2005,15000). The slope, using these two points, would be $m = \frac{15000 - 10000}{2005 - 2000} = \frac{5000}{5} = 1000$. We would say the average rate of change is 1000 persons per year. **The label is important**

in average rate of change. You will have the word per. In the example above, the slope means on average, the town grew by 1000 persons per year during that period.

Ex 11: Suppose a person receives a drug injected into a muscle. The concentration of the drug in the body, measured in milligrams per 100 milliliters is a function of the time elapsed after the injection. Suppose after 3 hours, there is 0.05 milligrams per 100 milliliters and after 7 hours, there is 0.02 milligrams per 100 milliliters. This data could be written as the ordered pairs (3, 0.05) and (7, 0.02). Find the average rate of change and describe what it means.