

An **exponential equation** is an equation with a variable in one of its exponents.

A **logarithmic equation** is an equation with one or more logarithmic expressions that contain a variable.

Solving an exponential equations:

A If you can write the left and right sides using the same base, then the exponents are the same. For example:

$$8^{3x} = 16^{5x-1}$$

$$(2^3)^{3x} = (2^4)^{5x-1}$$

$$2^{9x} = 2^{4(5x-1)}$$

$$9x = 20x - 4$$

$$4 = 11x$$

$$\frac{4}{11} = x$$

If  $b^M = b^N$ , then  $M = N$

If possible, try to write both exponential expression with the same base.

- 1) Write expression as  $b^M = b^N$
- 2) Set  $M = N$
- 3) Solve for the variable.

This cannot always be done, however.

If two quantities are equal, the logs to the same base of those quantities are equal.

Therefore:  $\log_b M = \log_b N \Leftrightarrow M = N$

B Therefore, to solve an exponential equation:

1. Write with exponential expression on one side.
2. Take either the common logs or the natural logs of both sides.
3. Use the properties of logs and solve.
4. Use a scientific calculator if asked to approximate.

Ex 1: Solve each equation. Approximate to 4 decimal places, if necessary.

a)  $8^{x-2} = 2^{2x}$

b)  $3^{x^2-2x} = \frac{1}{3}$

c)  $9^x = 13$

$$d) 10^{x-1} = 25$$

$$e) e^{3x-2} - 5 = 256$$

Solving logarithmic equations:

There are two types of logarithmic equations.

1. Type 1

- Express the equation in the form  $\log_b M = \log_b N$ , a single logarithm on each side. You may have to use the properties of logarithms.
- Use the 1-1 property ( If  $\log_b M = \log_b N$ , then  $M = N$  ). In other words, set the arguments equal.
- Solve for the variable.
- **Check** the proposed solution(s) in the original equation. Arguments should all be positive.

2. Type 2

- Express the equation in the form  $\log_b M = c$ .
- Rewrite the equation in exponential form  $b^c = M$
- Solve for the variable.
- **Check** all proposed solution(s) in the original equation. Arguments should all be positive.

The coefficient of each logarithm should be one.

When solving logarithmic equations, always remember that **any value for the variable must make any arguments be positive**. A logarithm of a negative number does not exist. Any possible solution that makes a 0 or negative argument must be disregarded.

Ex 2: Solve each of these equations.

a)  $\log_2(x-4) - \log_2(3x-10) = -\log_2 x$

b)  $\ln(3x + 2) = \ln(4x + 10)$

c)  $\ln(x - 3) = \ln(7x - 23) - \ln(x + 1)$

Ex 3: Solve each equation.

a)  $2\log_3(7 + x) = 4$

b)  $\log x + \log(x + 9) = 1$

c)  $2\log_3 x - \log_3(x-4) - \log_3 2 = 2$

d)  $\log x = 1 - \log(x-9)$

e)  $\log \frac{5x+2}{2(x+7)} = 0$