If a parabola opens downward, the vertex is the highest point of the graph. The y-coordinate of that vertex is the largest value in the range of the function. **That y-coordinate value is called the maximum of the function** or equation and that maximum occurs when $x = h$. There are several real life applications where a maximum value can be found by writing an equation for a parabola and finding its vertex.

It is important when solving these applied problems that you understand what that the h of the vertex represents the x-coordinate and the k of the vertex represents the y-coordinate. Examine the following quadratic functions.

$P(x) = -0.02x^2 + 4x + 12$ This function represents a profit of a company that produces and sells x items. Each ordered pair is of the form (x, P) or (# items, profit)
After the vertex (h, k) is found, the h represents the number of items (x) made and sold to get a maximum profit. The k (y or $P(x)$)is that maximum profit.

$A(w) = -2w^2 + 650w$ This is an area of a region as a function of its width w, where
$L = 650 - 2w$ Each ordered pair is of the form (w, A) or (width, area).
After the vertex (h, k) is found, the h represents the width that gives a maximum area. (Length could then be found by solving for L.) The k represents the maximum area.

Ex 1:   A room in a home has a parabolic doorway with an equation
$h = h(x) = -\dfrac{1}{100}x^2 + 2$ where x is horizontal distance (in meters) from the center
of the doorway and h is height in meters. Find the maximum height of the doorway.

(0,0)
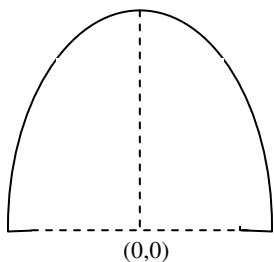
Ordered pairs are of the form (x, H) or (horiz. d,height). $h =$ horizontal distance from center of base and $k =$ maximum height.

Ex 2: A gardener has 100 feet of fencing for a rectangular flower garden. Write an
equation for the area of the garden. Find the dimensions of the garden that would
give a maximum area. What is that area?

We need to write an area
equation as a function of either
length or width. Ordered pairs
will be of the form (w or L, A).
The h of the vertex represents
the width or length and the k of
the vertex represents the
maximum area of the garden.

Ex 3: A farmer has 1200 feet of fencing for a rectangular area adjacent to a barn , where
the barn will be used as one side of the area. (See the diagram.)   Assume the
barn is as long or longer than the length parallel to it.

barn
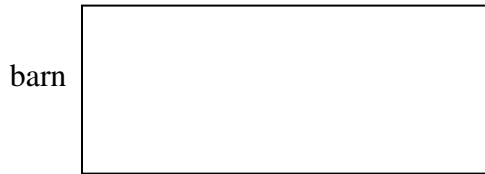
Write an equation for the total area. What dimensions of the area would give a
maximum total area?

Ex 4: A hotel with 200 rooms is filled every night when the room rate is $90. Experience has shown that for every $5 increase in cost, 10 fewer rooms will be occupied. Let n = number of $5 increases in room cost and write an equation to represent the nightly income from the rooms. Find the cost per room that will make the income a maximum? What is that maximum income?

# inc.	cost/room	# rooms
0	90	200
1	95	190
2	100	180
n		

Write a function for income based on the number of $5 increases in room cost.

Ex 5: A local store that sells deli sandwiches has weekly fixed costs of $650, and variable costs for making each sandwich of $0.90 per sandwich. The weekly revenue that the store collects is given by $R(x) = -0.003x^2 + 6x$, where x is the number of deli sandwiches made and sold.
Write a profit function to represent the weekly profit of the store as a function of the number of sandwiches sold. How many sandwiches have to be made and sold in order for the store to have a maximum profit? What is that maximum profit?

Ex 6: A rocket is fired upward from a point well above the ground. Its height in feet above the ground after t seconds is given by $h = -16t^2 + 48t + 288$. In how many seconds will the rocket reach its maximum height? What is that maximum height? When will the rocket hit the ground?

Ex 7: A bus on a route between two cities charges of fare of $80 per person plus $5 per person for each unsold bus seats. If the bus holds 40 passengers, let x represent the number of unsold seats and write a function to represent the total revenue received for the bus trip.

Find the number of unsold seats that result in maximum revenue and what that revenue would be.

A parabola's standard equation can be found given the coordinates of the vertex and another point of the parabola. The value of a must be found by substituting the coordinates of the vertex and point in standard form $y = a(x-h)^2 + k$.

Ex8:   Find the equation in standard form for each parabola described.
    a)   vertex : $(2,5)$ point : $(0,0)$

    b)   vertex : $(-2,2)$, point : $(4,-7)$

Ex 9:   A cannonball follows the parabolic trajectory path and reaches a height of 225 feet. The cannon is located at the origin (0, 0). There is a castle is 35 feet away from the cannon. If the cannonball falls 5 feet short of the castle, find an equation for the path of the cannonball.