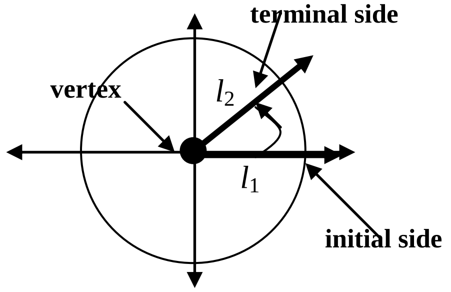
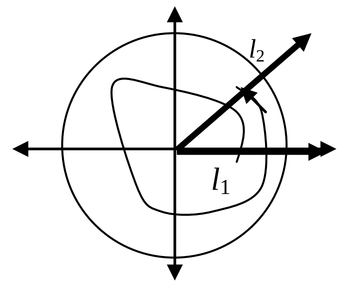
Section 6.1 **Angles**

DEFINITIONS:
An **angle** is defined as the set of points determined by two rays, or half-lines, l_1 and l_2 having the same end point *O*. An angle can also be considered as two finite line segments with a common point.

We call l_1 the **initial side**, l_2 the **terminal side**, and O the **vertex** of angle ∠AOB. The direction and number of rotations of l_1 makes before stopping at l_2 is not restricted. If two angles have the same initial and terminal sides, they are **coterminal angles**.



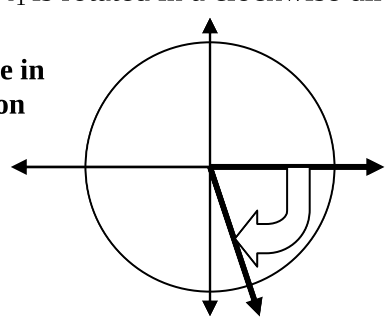


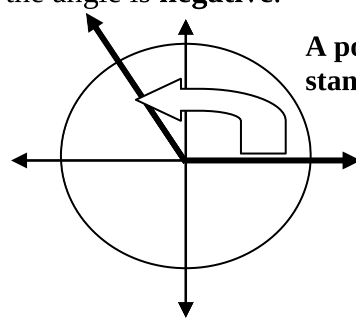
coterminal angles

A **straight angle** is an angle whose sides lie on the same straight line but extend in opposite directions from its vertex.

If we introduce the rectangular coordinate system, then the **standard position** of an angle is obtained by placing the vertex at the origin and letting the initial side l_1 coincide with the positive x-axis. If l_1 is rotated in a counterclockwise direction to the terminal position l_2, then the angle is considered **positive**. If l_1 is rotated in a clockwise direction, the angle is **negative**.

A negative angle in standard position





A positive angle in standard position

An angle is called a **quadrantal angle** if its terminal side lies on a coordinate axis.

One unit of measurement for angle is the **degree**. The angle is standard position obtained by one complete revolution in the counterclockwise direction has measure 360 degrees, written 360°

Two coterminal angles will differ by multiples of 360°. To find a coterminal angle, add or subtract multiples of 360°.

Find two positive coterminal angles and two negative coterminal angles.

ex. $\theta = 55°$

ex. $\theta = -100°$

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A **right angle** is half of a straight angle and has measure 90°

An **acute angle** θ; $0° < \theta < 90°$

An **obtuse angle** θ; $90° < \theta < 180°$

Complementary angles α, β; $\alpha + \beta = 90°$

Supplementary angles α, β; $\alpha + \beta = 180°$

For smaller units than degrees we have two choices:

1) tenths, hundredths, thousandths, and ten- thousandths of a degree. example: 150.1234°
2) divide the degrees into 60 equal parts, called **minutes** (denoted by ') and each minute into 60 equal parts, called **seconds** (denoted by ") example: 50° = 49° 59' 60"

ex. Find the angle that is complementary to θ = 45.7°

ex. Find the angle that is supplementary to θ = 155.41°

ex. Find the angle that is complementary to θ = 39° 34' 19"

ex. Find the angle that is supplementary to θ = 52° 13' 45"

To convert from a decimal part of a degree to minutes and seconds: Multiply the decimal part of the degree by 60 minutes/degree. The result is the number of minutes. Take the decimal part of a minute and multiply by 60 seconds/minute and round to nearest number of seconds. Multiply by the ratios $\frac{60'}{1^\circ}$ or $\frac{60''}{1'}$ **. This is DD → DMS!**

Express the angle in terms of degrees, minutes and seconds.

ex. θ = 150.1459°

ex. θ = 12.6789°

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To convert from minutes and seconds to decimal part of a degree: Write x minutes as x/60 of a degree and y seconds as y/3600 of a degree, convert to decimals, add and round.

Multiply by the ratios $\frac{1^\circ}{60'}$ or $\frac{1^\circ}{3600''}$ **This is DMS→DD**

Express the angle as a decimal to the nearest ten-thousandth.

ex. $\theta = 45° 16' 45''$

ex. $\theta = 115° 50' 12''$

Another unit of measurement for angles is the **radian**. One radian is the measure of the central angle of a circle **subtended** by an arc equal in length to the radius of the circle. Since the circumference of a circle is $2\pi r$, the number of times r units can be 'laid off' around the circle is 2π.

********Thus 360° = 2π radians and therefore 180° = π radians**. ******

When the radian measure of an angle is used, no units will be indicated.
$\theta = 5$ means $\theta = 5$ radians. $\Theta = 5°$ means 5 degrees.

Since $\boxed{180° = \pi \text{ radians}}$, use the ratios $\frac{\pi \text{ radians}}{180°}$ and $\frac{180°}{\pi \text{ radians}}$ (since both = 1) to covert degree measures to radians and radian measures to degrees. To remember which ones to use, think of which unit needs to be canceled and which unit is to be left as the label.

Multiply by $\frac{\pi}{180°}$ to convert degrees to radians. Multiply by $\frac{180°}{\pi}$ to convert radians to degrees.

Find the exact radian measure.

ex. 30° ex. 60° ex. 90° ex. 120°

ex. -45° ex. 250° ex. 450° ex. 360°

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Find the exact degree measure.

ex. $\frac{3\pi}{4}$ ex. $\frac{5\pi}{3}$ ex. $\frac{3\pi}{2}$ ex. $\frac{11\pi}{6}$

Approximate the number of degrees to 3 decimal places.
ex. 3 radians

Express the angle in terms of degrees, minutes and seconds.

ex. $\theta = 3$ ex. $\theta = 5.6$

Two coterminal angles will differ by multiples of 2π radians. To find a coterminal angle, add or subtract multiples of 2π.

Find two positive coterminal angles and two negative coterminal angles.

ex. $\theta = \frac{\pi}{3}$ ex. $\theta = \frac{7\pi}{6}$ ex. $\theta = -\frac{5\pi}{4}$

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Remember: Coterminal angles vary by multiples of 360°. Since 360° equals 2π, to find a coterminal angle that is in radian measure, add or subtract multiples of 2π.

Remember, when converting from:

degrees to radians: multiply by $\frac{\pi}{180°}$ radians to degrees: multiply by $\frac{180°}{\pi}$

Express θ in terms of degrees, minutes and seconds.

$\theta = 2.3$

Find the exact radian measure of θ

$\theta = 240°$

Find the length of arcs and the area of a sector:

If θ **is in radians** then the length of the arc created by θ is found by: $s = r\theta$ (From $C = 2\pi r$)

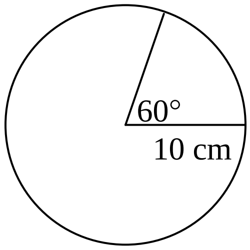
$C = 2\pi r$

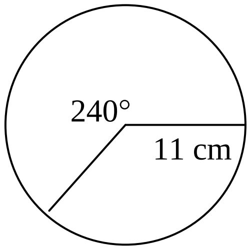
For a circle: $C = \theta = 2\pi$

Therefore: $C = \theta r$

which means in general $s = \theta r$

Find the length of the arc subtended by the central angle.





If θ is in radians then the area of the sector created by θ is found by: A = $\frac{1}{2}r^2\theta$ (From A = πr^2)

$$A = \pi r^2$$

Since $\theta = 2\pi$ for a circle

$$\frac{\theta}{2} = \pi$$

Substitute: $A = \left(\frac{\theta}{2}\right)r^2$

or $A = \frac{1}{2}r^2\theta$

Find the area of the sectors subtended by the central angle in the examples at the bottom of page 5.

Find the radian and degree measures of the central angle θ subtended by the given arc of length s on a circle of radius r.

a) Find the radian and degree measures (DD and DMS) of the central angle θ.
b) Find the area of the sector determined by θ

s = 8 cm, r = 3 cm

s = 6 ft, r = 18 in.

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a) Find the length of the arc that subtends the given central angle θ on a circle of diameter d.
b) Find the area of sector determined by θ

$\theta = 50°$, $d = 16$ m

$\theta = 3.1$, $d = 44$ cm

The minute hand of a clock is 6 inches long. How far does the tip of the minute hand move in 15 minutes? How far does it move in 25 minutes?