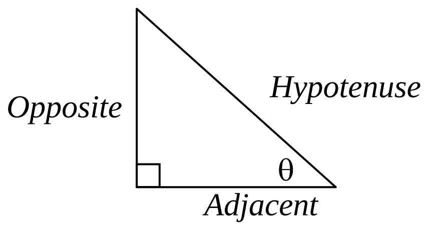
MA 15400 Lesson 2 Summer Section 6.2

Trigonometric Functions of Angles

We will introduce the trigonometric functions in the manner in which they originated historically- as ratios of sides of a right triangle.

A triangle is a right triangle if one of its angles is a right angle.



If θ would be changed to the other acute angle of the triangle, the opposite and adjacent sides would switch. **Labeling triangles is always important. Changing θ changes the adjacent and opposite sides.** The hypotenuse is always the same side so matter where θ is put.

With θ as the acute angle of interest, the adjacent side is abbreviated adj., the opposite side is abbreviated opp., and the hypotenuse is abbreviated hyp. With this notation, the six trigonometric functions become:

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

$$\csc\theta = \frac{hyp}{opp} \qquad \sec\theta = \frac{hyp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

The sine, cosine, and tangent are the **3 major or primary trigonometric functions.** The cosecant, secant, and cotangent functions are the lesser or minor functions.

SOH-CAH-TOA
sine is opposite over hypotenuse
cosine is adjacent over hypotenuse
tangent is opposite over adjacent

Notice that the cscθ is the reciprocal of sinθ, secθ is the reciprocal of cosθ, and cotθ is the reciprocal of tanθ.

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

These are called the RECIPROCAL IDENTITIES.

Since the hypotenuse is always the largest side of the triangle,

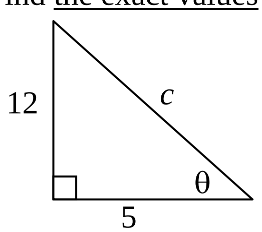
$0 < \sin\theta < 1 \qquad 0 < \cos\theta < 1$

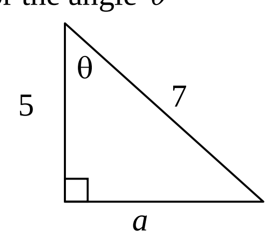
(in a right triangle)

$\csc\theta > 1 \qquad \sec\theta > 1$

Because the hypotenuse is always the largest side, any of the functions with hyp in the denominator, will be less than 1. Any with hyp in the numerator will be greater than 1.

Find the exact values of the six trigonometric functions for the angle θ





Trigonometric Functions of Angles

Find the exact values of the six trigonometric functions for the acute angle θ

$\cos\theta = \frac{9}{41}$ $\cot\theta = \frac{35}{12}$ $\csc\theta = 5$

$\sec\theta = \frac{5}{2}$ $\sin\theta = \frac{7}{3}$ $\tan\theta = \frac{195}{28}$

Using your calculator, find (approximate to 4 decimal places):

DEGREES

sin(134°) cos(-54°) tan(121°)

sec(-94°) csc(25°) cot(330°)

RADIANS

sin(5) cos(-0.123) $\tan\left(\frac{6}{5}\right)$

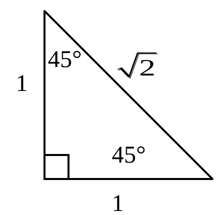
Trigonometric Functions of Angles

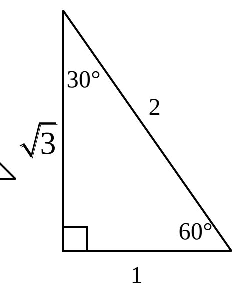
sec(3.1π) csc(5.6) cot(-9)

This is the table of trigonometric values you should be able to recall or derive.

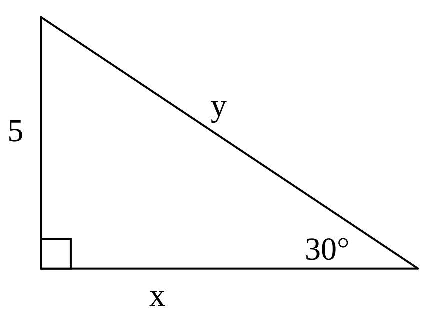
	ANGLES	θ	θ	θ
	Degrees	30°	45°	60°
	Radians	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
	$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{2}\right)$	$\frac{\sqrt{3}}{2}$
Reverse the sine row	$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{2}\right)$	$\frac{1}{2}$
Divide sine row by cosine row	$\tan\theta$	$\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{3}\right)$	1	$\sqrt{3}$

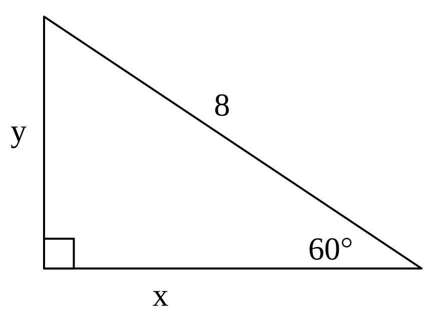
This table can easily be verified by examining a 45-45 right triangle and a 30-60 right triangle.

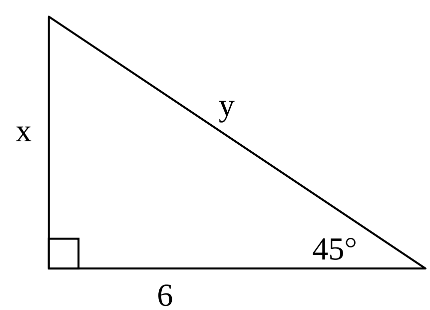




Find the **exact** values of x and y.







Trigonometric Functions of Angles

Always draw a picture with applied problems!

A building is known to be 500 feet tall. If the angle from where you are standing to the top of the building is 30°, how far away from the base of the building are you standing?

The angle to the top of a flag pole is 41°. If you are standing 100 ft. from the base of the flagpole, how tall is the flagpole?

The angle to the top of a cliff is 57.21°. If you are standing 300 m from a point directly below the cliff, how high is the cliff?