Vectors

In the previous lesson we discussed unit vectors on the positive *x*-axis (*i*) and on the positive *y*-axis (*j*). What is we wanted to find other unit vectors? There are an infinite number of unit vectors in all directions from the origin.

A unit vector in the **same direction** as a vector *a* would be $\left(\frac{1}{\|a\|}\right)a$ and a unit vector in the **opposite direction** as a vector *a* would be $-\left(\frac{1}{\|a\|}\right)a$. *u* is often used for a unit vector.

(Basically, the vector is divided by its length, leaving a length of 1.)

Ex 1) If vector $a = 6i - 7j$, find a **unit** vector that has…

(a) the same direction as *a*.

(b) the opposite direction as *a*.

Ex 2) If vector $b = \langle 9, -4 \rangle$, find a vector that has…

(a) twice the magnitude of *b* in the same direction as *b*.

(b) one-half the magnitude of *b* in the same direction as *b*.

(c) twice the magnitude of *b* in the opposite direction as *b*.

(d) one-half the magnitude of *b* in the opposite direction as *b*.

You will notice each of these vectors has the same slope.

Remember that slope is the tangent value.

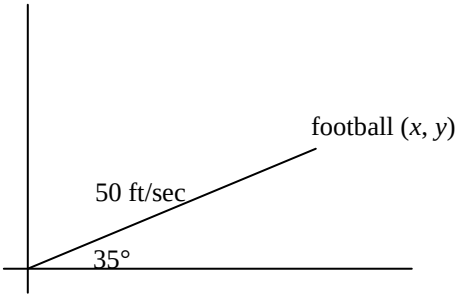
The slope of each of these vectors is $-\frac{4}{9}$.

Ex 3) If vector $c = 5i - 12j$, find a vector of magnitude 5 that has…

(a) the same direction as c.

(b) the opposite direction as c.

4) A quarterback releases a football with a velocity of 50 ft/sec at an angle of 35° with the horizontal. Approximate the **horizontal and vertical components** of the vector.



horizontal component:

vertical component:

In the last lesson, we had problems where the components of the vector were given and the magnitude and angle θ (from positive *x*-axis) were found. In this problem, we are given the angle and magnitude and asked to find the components.

$\|football\| =$

check:

$\tan\theta = \frac{opp}{adj} =$

Summary:

$x = a_1 = \|a\|\cos\theta$ and $y = a_2 = \|a\|\sin\theta$

for the components $\langle a_1, a_2 \rangle$

Note: θ is always the angle from the positive *x*-axis.

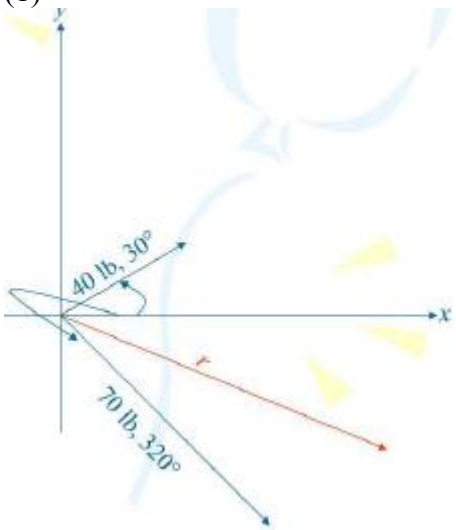
5) The magnitudes and directions of two forces acting at a point P are given in (a) and (b). Approximate the **magnitude** and **direction** of the resultant vector.

(a) 70 lb, 320° (b) 40 lb, 30°

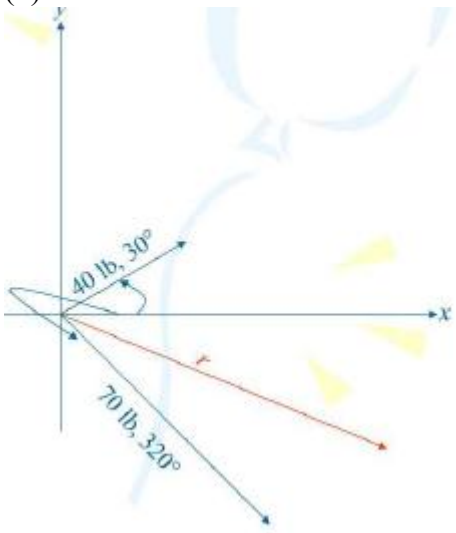
This problem is similar to problems from the previous lesson, except the angle between the vectors will have to be determined.

There are two method to work these types of problems.
(1) Using a parallelogram and the law of cosines and law of sines
(2) Using the sum of the components and a tangent function

(1)



(2)



components of vector 1:

components of vector 2:

components of resultant *r*:

6) The magnitudes and directions of two forces acting at a point P are given in (a) and (b). Approximate the **magnitude** and **direction** of the resultant vector.

(a) 20 kg, S17°W (b) 50 kg, N82°W

Picture:

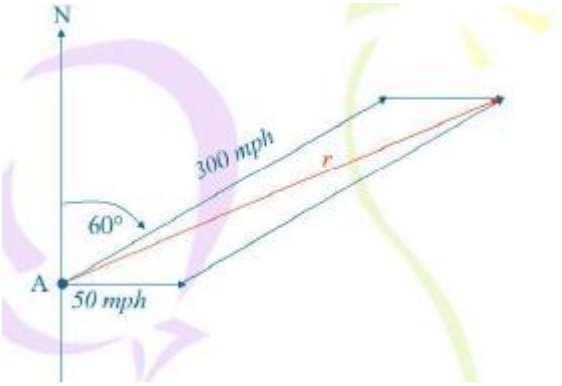
(resultant vector always begins at origin)

(1) Parallelogram method

(2) Component method Remember: The angle is always from the positive *x*-axis.

7) An airplane with airspeed of 300 mph is flying in the direction 60° and a 50 mph wind is blowing directly from the west. The direction of p + w gives the **true course** of the airplane relative to the ground, and ||p + w|| is the **ground speed** of the airplane. Approximate the true course and ground speed.

air speed and direction = airplane (pilot) and wind
true course and ground speed = resultant



8) An airplane pilot wishes to maintain a true course in the direction 300° with a ground speed of 450 mph when the wind is blowing directly north at 60 mph. Approximate the required airspeed and compass heading.

This problem gives the resultant and the wind. We need to find information about the plane.

9) For a motorboat moving at a speed of 40 mph to travel directly north across a river, it must aim at a point that has the bearing N20°E. If the current is flowing directly west, approximate the rate at which it flows.