MA 154 Lesson 22 Delworth

Section 8.4 The Dot Product

The dot product of two vectors has many applications. We begin with an algebraic definition:

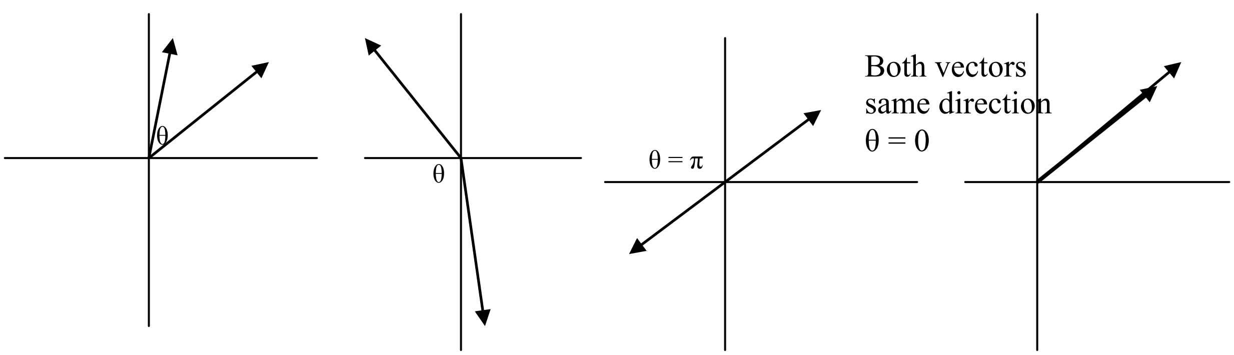
Let $a = \langle a_1, a_2 \rangle = a_1 i + a_2 j$ and $b = \langle b_1, b_2 \rangle = b_1 i + b_2 j$

The dot product of *a* and *b*, denoted $a \cdot b$ is: $a \cdot b = \langle a_1, a_2 \rangle \cdot \langle b_1, b_2 \rangle = a_1 b_1 + a_2 b_2$ **The dot product is a number, not a vector!** The dot product is the sum of the product of the x's and the product of the y's.

1) Find the dot product of the two vectors. $a = \langle -2, 5 \rangle$, $b = \langle 3, 6 \rangle$

2) Find the dot product of the two vectors. $a = 4i - 6j$, $b = 3i + 2j$

Any two nonzero vectors *a* and *b* may be represented in a coordinate plane by directed line segments from the origin *O* to the point $A(a_1, a_2)$ and $B(b_1, b_2)$, respectively. The angle θ between *a* and *b* is, by definition, ∠AOB. Note that $0 \le \theta \le \pi$ and that $\theta = 0$ if *a* and *b* have the same direction or $\theta = \pi$ if *a* and *b* have opposite directions.



Summary:
1) The angle between any two vectors θ will be such that $0 \le \theta \le \pi$
2) If the angle is 0, the vectors are in the same direction.
3) If the angle is π, the vectors are in opposite directions.

Definition of Parallel and Orthogonal Vectors:

Let θ be the angle between two nonzero vectors *a* and *b*:

(1) *a* and *b* are parallel if θ = 0 or θ = π
(vectors are in the same or opposite directions)

(2) *a* and *b* are orthogonal if $\theta = \frac{\pi}{2}$
(vectors are perpendicular)

Theorem on the Dot Product:
If θ is the angle between two nonzero vectors *a* and *b*, then: $a \cdot b = \|a\|\|b\|\cos\theta$

From the theorem:

$\cos\theta = \frac{a \cdot b}{\|a\|\|b\|}$ This is the one we give on the formula sheet.

The dot product and the magnitudes of the vectors can help find the angle between the two vectors.

. 3) Find the angle between the two vectors.

$a = 8i - 3j,\ b = 2i - 7j$

4) Find the angle between the two vectors.

$a = 3j,\ b = 2i + 6j$

Two vectors *a* and *b* are orthogonal if and only if $a \times b = 0$

Suppose two vectors are perpendicular and form a 90˚angle.

Proof: $\cos 90^\circ = 0$

$$\cos 90^\circ = \frac{a \cdot b}{\|a\|\|b\|} \rightarrow 0 = a \cdot b$$

6) Show that the vectors are parallel, and determine whether they have the same direction or opposite directions.

a) $a = \frac{5}{3}i + 10j, \quad b = 3i + 18j$

b) $a = \langle 1, -4 \rangle, \quad b = \langle -2, 8 \rangle$

5) Show that the two vectors are orthogonal.

a) $a = \langle 4, -1 \rangle, \ b = \langle 2, 8 \rangle$

b) $a = 4i - 6j, \ b = 3i + 2j$

7) Determine m such that the two vectors are orthogonal.

a) $a = 3i - 2j, \quad b = 4i + 5mj$

b) $a = 9i - 16mj, \quad b = i + 4mj$

Dot product

Let $a = \langle a_1, a_2 \rangle = a_1 i + a_2 j$ and $b = \langle b_1, b_2 \rangle = b_1 i + b_2 j$

The **dot product** of *a* and *b*, denoted $a \cdot b$, is:

$$a \cdot b = \langle a_1, a_2 \rangle \cdot \langle b_1, b_2 \rangle = a_1 b_1 + a_2 b_2$$

Any two nonzero vectors *a* and *b* may be represented in a coordinate plane by directed line segments from the origin *O* to the point $A(a_1, a_2)$ and $B(b_1, b_2)$, respectively. The angle θ between *a* and *b* is, by definition, ∠AOB.

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Definition of Parallel and Orthogonal Vectors:

Let θ be the angle between two nonzero vectors *a* and *b*:

(1) *a* and *b* are parallel if $\theta = 0$ or $\theta = \pi$

(2) *a* and *b* are orthogonal if $\theta = \dfrac{\pi}{2}$

Theorem on the Dot Product:

If θ is the angle between two nonzero vectors *a* and *b*, then: $a \cdot b = \|a\|\|b\|\cos\theta$

From this theorem:

$$\cos\theta = \frac{a \cdot b}{\|a\|\|b\|}$$

This is the one we give on the formula sheet.

Two vectors *a* and *b* are **orthogonal** if and only if $a \cdot b = 0$