MA 15400 Lesson 24 Section 4.5

Rational Functions

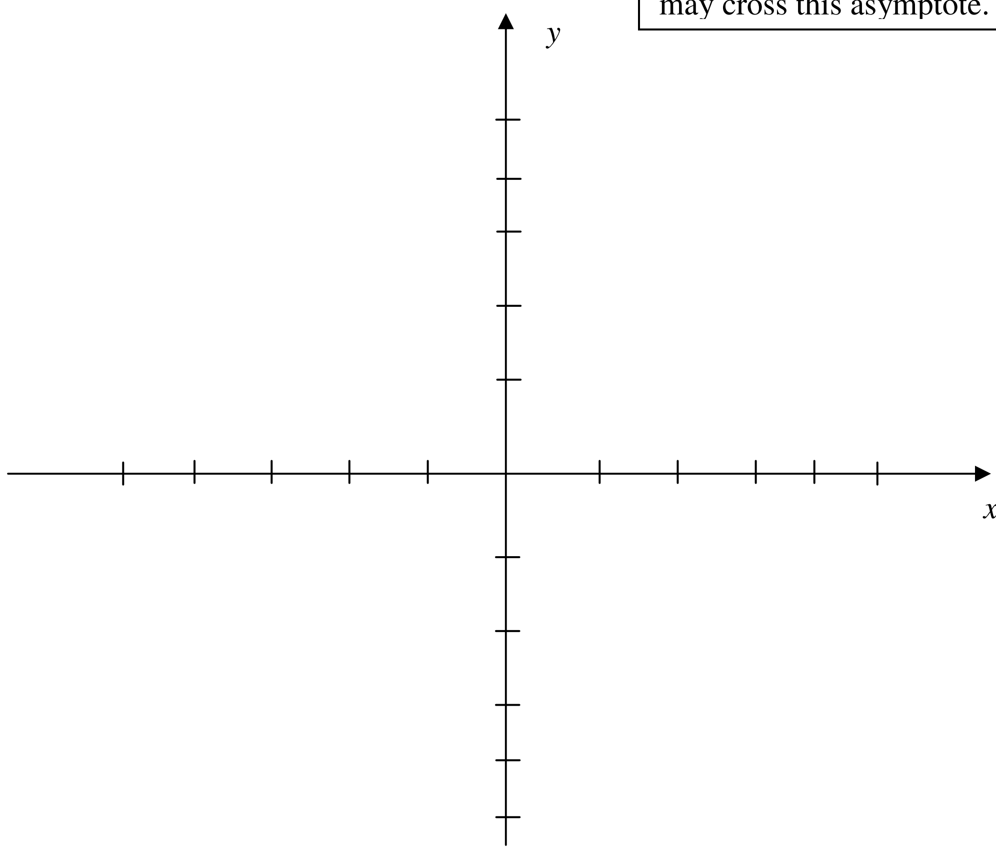
Steps for sketching the graph of rational functions:

1) Find the horizontal asymptote, if it exists.
2) Find the vertical asymptote(s), if any exist.
3) Find the y-intercept, if it exists.
4) Find the x-intercept(s), if any exist.
5) **Does the graph cross the horizontal asymptote? If the answer is yes, where does it cross?**
6) Sketch the function.

If the horizontal asymptote is the x-axis, then we will know if and where the graph crosses the asymptote; at the x-intercepts. If the horizontal asymptote exists and is not the x-axis, we need to find the point where the graph may cross this asymptote.

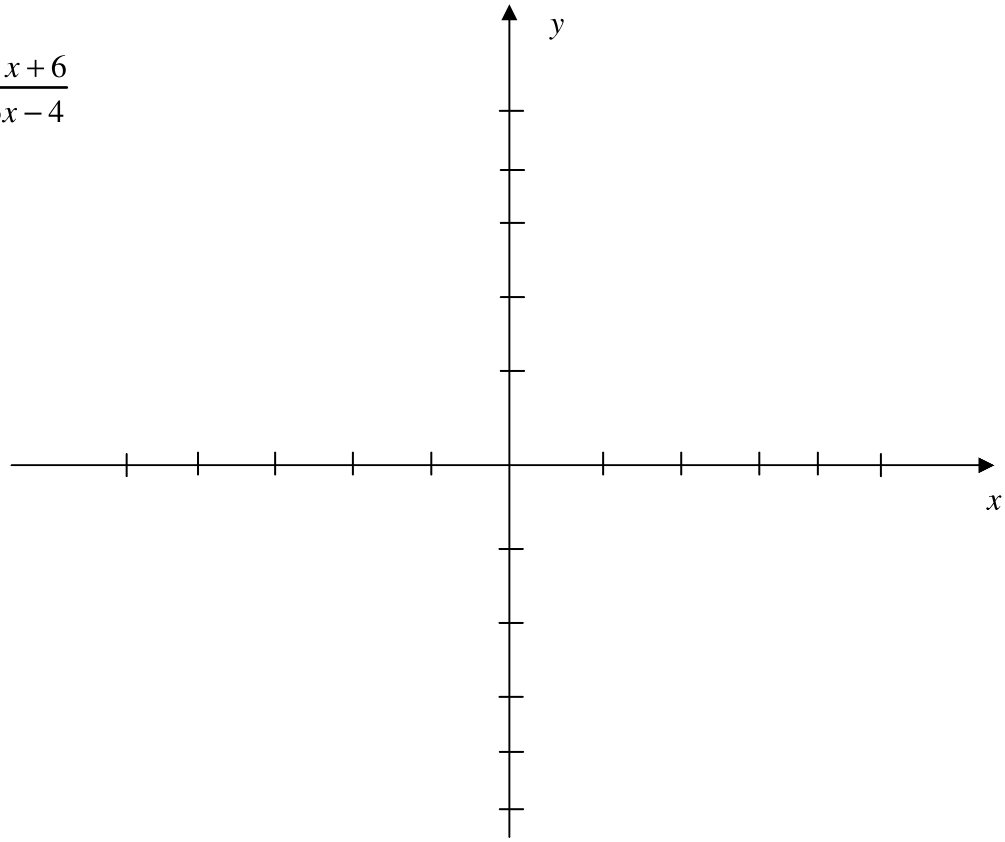
Sketch:

$$f(x)=\frac{2x^2-2x-4}{x^2+x-12}=\frac{2(x-2)(x+1)}{(x+4)(x-3)}$$

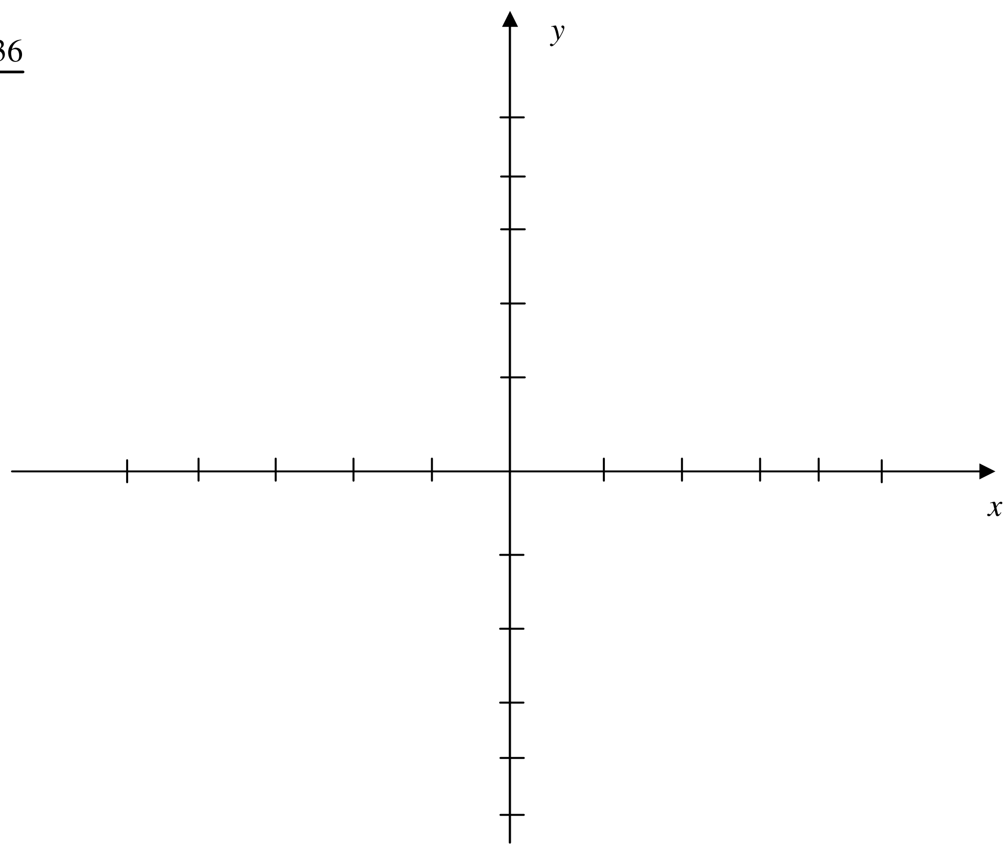


Rational Functions

$$f(x) = \frac{-x^2 - x + 6}{x^2 + 3x - 4}$$



$$f(x) = \frac{3x^2 - 3x - 36}{x^2 + x - 2}$$



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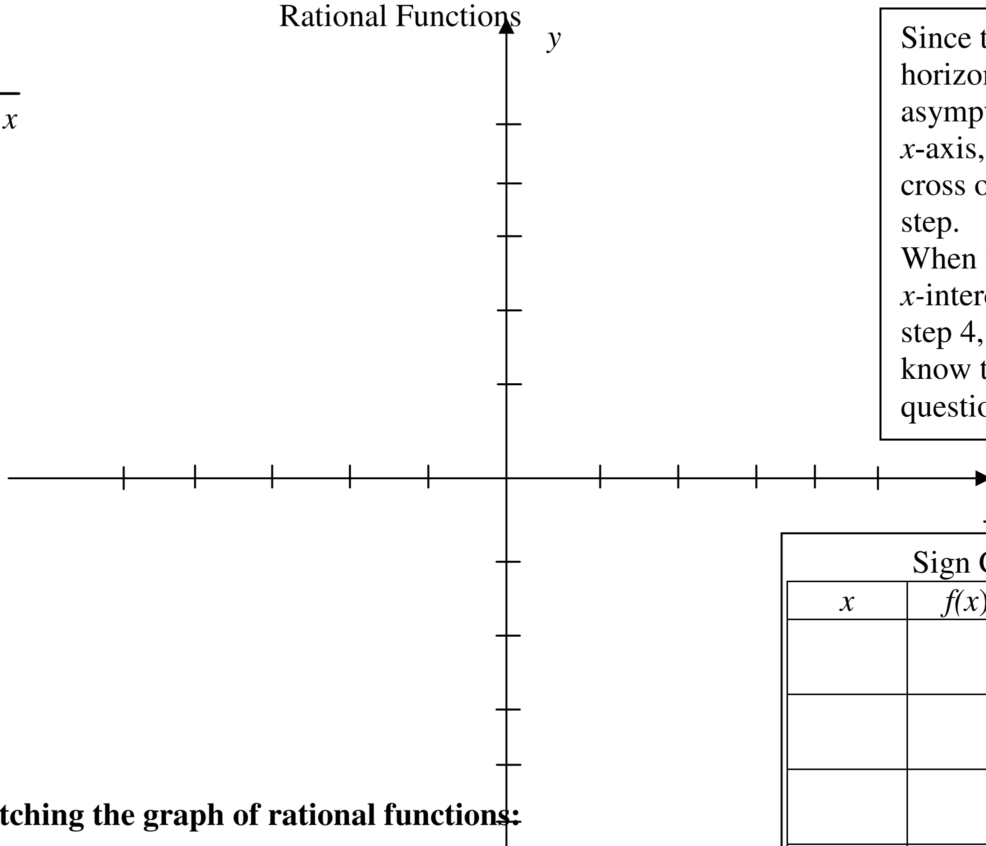
Rational Functions

$f(x)=\dfrac{x-1}{x^3-4x}$



Since the horizontal asymptote is the *x*-axis, we can cross out the fifth step.
When we find the *x*-intercepts in step 4, we will know that question.

Sign Chart

x	*f*(*x*)	Sign

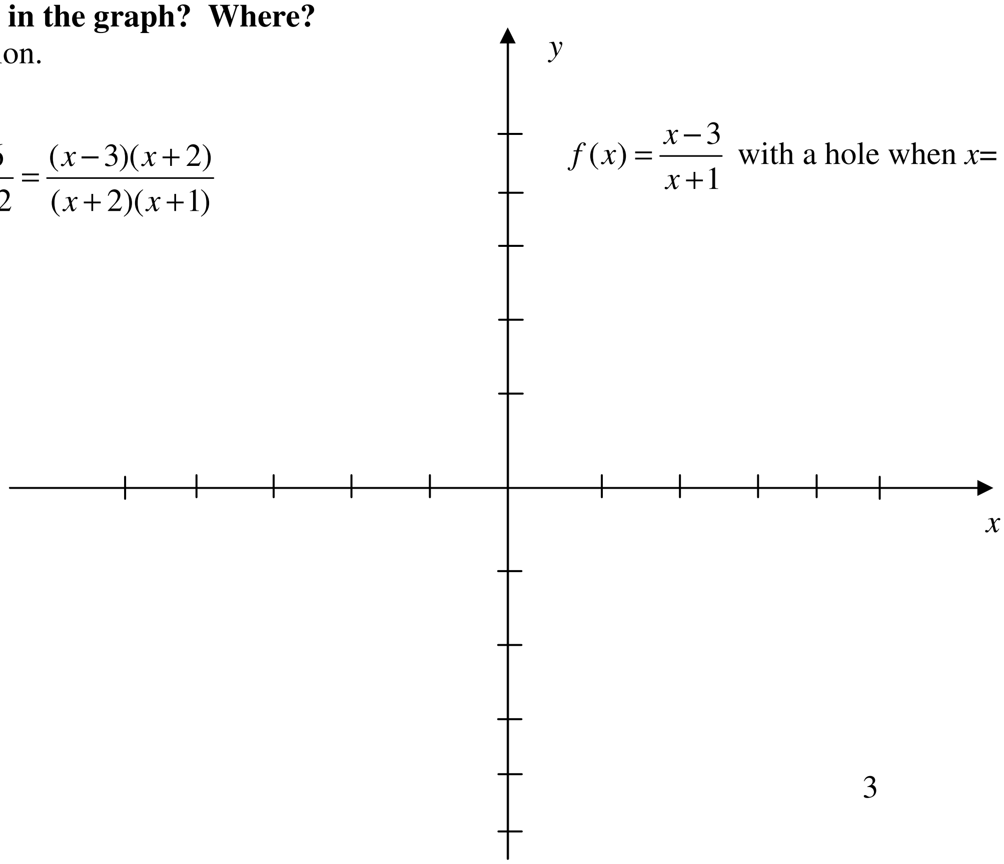
Steps for sketching the graph of rational functions:

1) Find the horizontal asymptote, if it exists.
2) Find the vertical asymptote(s), if any exist.
3) Find the *y*-intercept, if it exists.
4) Find the *x*-intercept(s), if any exist.
5) Does the graph cross the horizontal asymptote? If the answer is yes, where does it cross?
6) **Is there a 'hole' in the graph? Where?**
7) Sketch the function.

$f(x)=\dfrac{x^2-x-6}{x^2+3x+2}=\dfrac{(x-3)(x+2)}{(x+2)(x+1)}$

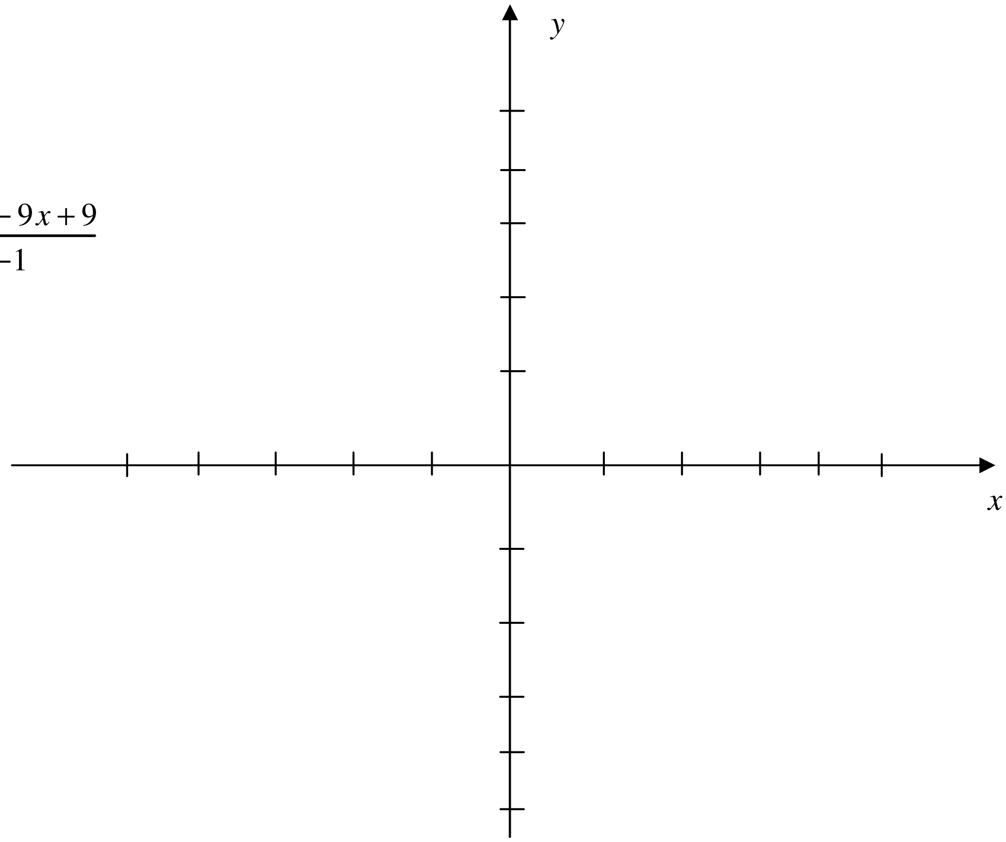
$f(x)=\dfrac{x-3}{x+1}$ with a hole when $x = -2$

When there is a common factor in both numerator and denominator, the graph will look similar to the graph as if that factor was cancelled. However, there will be a 'hole' at the point associated with that *x*-value, which makes a zero in the **original function.**. Always put an open circle at such points.

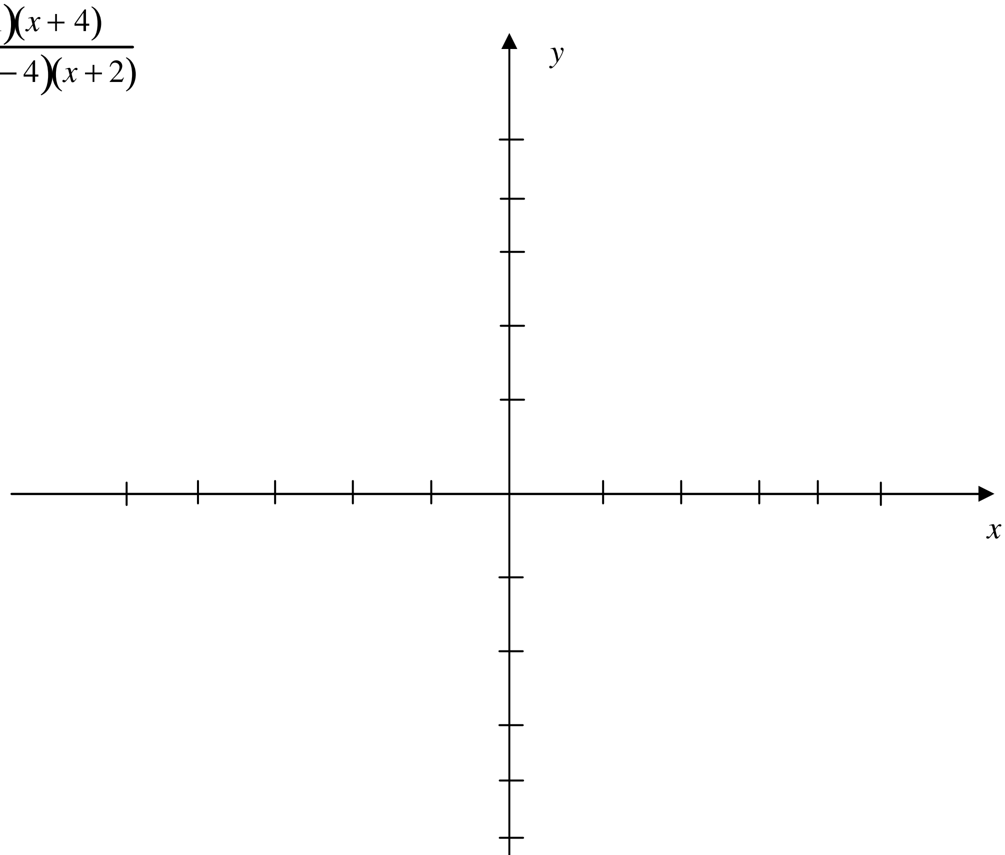


Rational Functions

$$f(x)=\frac{x^3-x^2-9x+9}{x-1}$$



$$f(x)=\frac{(x^2+1)(x+4)}{(x^2+3x-4)(x+2)}$$



Rational Functions

Find the equation of the rational function that satisfies the following conditions:

Vertical asymptote: x = -5
Horizontal asymptote: y = -2
x-intercept: 4

Vertical asymptotes: x = 3, x = -2
Horizontal asymptote: y = 0
x-intercept: -9, f(2) = -11

$$f(x)=\frac{a(x+9)}{(x-3)(x+2)}$$

Vertical asymptotes: x = 5, x = -2
Horizontal asymptote: y = -3
x-intercepts: -6, 7
Hole at x = 1

Vertical asymptotes: x = 4, x = -1
Horizontal asymptote: y = 0
x-intercept: 2
Hole at x = – 9
f(5) = 3