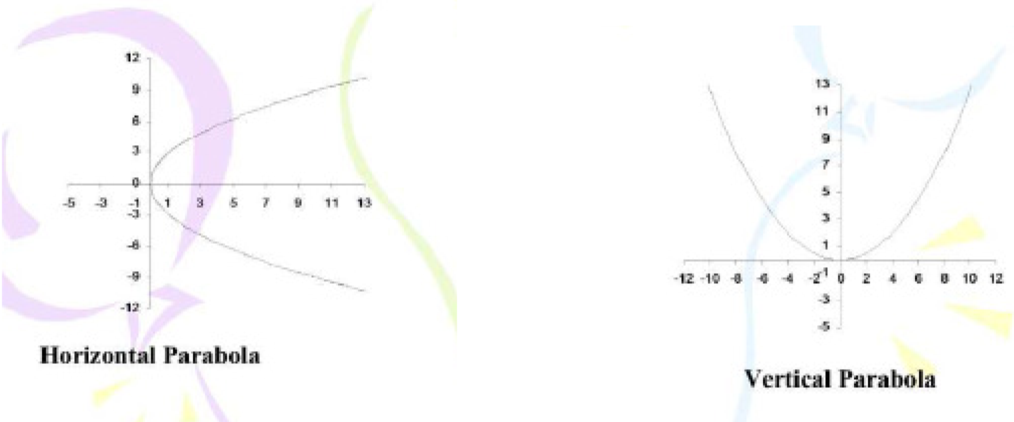
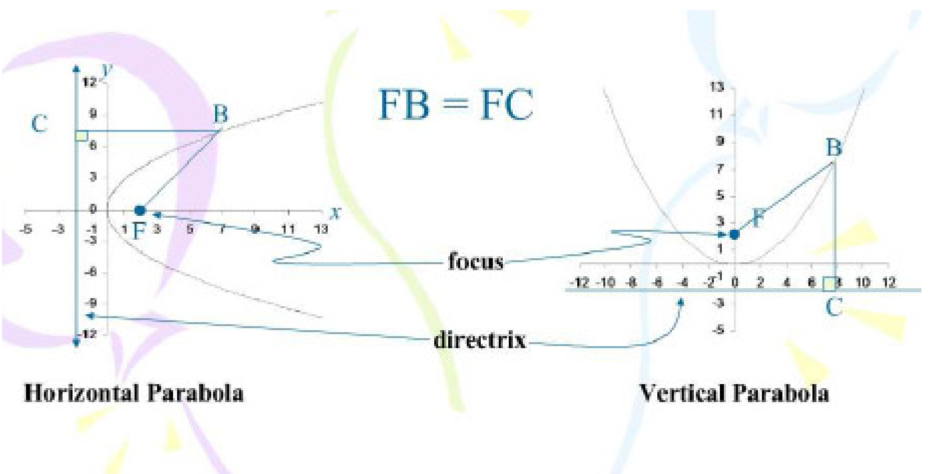
Parabolas



The graphs above represent **parabolas.** A **horizontal parabola** will open left or right. A **vertical parabola** opens up or down.

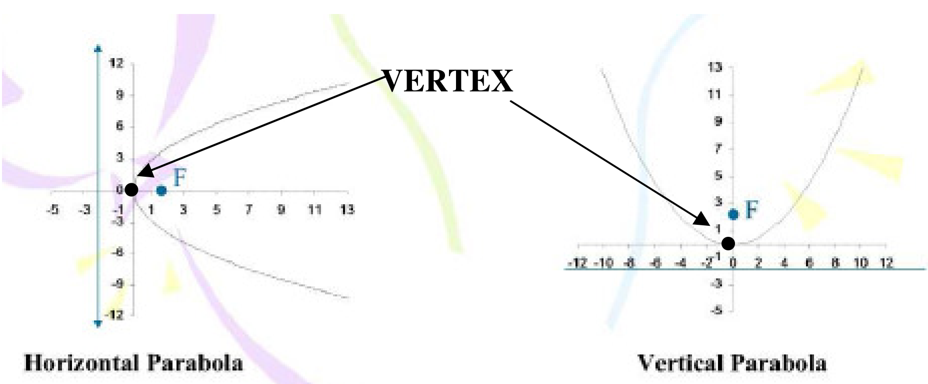
A **parabola** is the set of all points in a plane (only 2 dimensions) equidistant from a fixed point *F* (the **focus**) and a fixed line *l* (the **directrix**) that lie in the plane.



The fixed point, the focus, will always be 'inside' the parabola and the fixed line, the directrix, will always be 'outside' of the parabola. Every point on the parabola, represented by B in the picture, the distance from B to the focus F is equal to the distance from B perpendicular to the line at point C.

The **axis** of the parabola is the line through *F* that is perpendicular to the directrix.

The **vertex** of the parabola is the point *V* on the axis halfway from *F* to *l*. The vertex is the point on the parabola that is closest to the directrix.



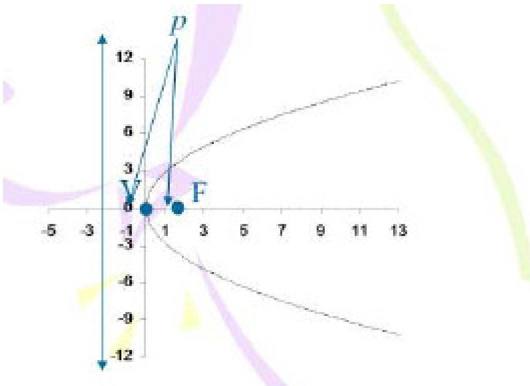
A horizontal parabola has a horizontal axis. A vertical parabola has a vertical axis.

Parabolas

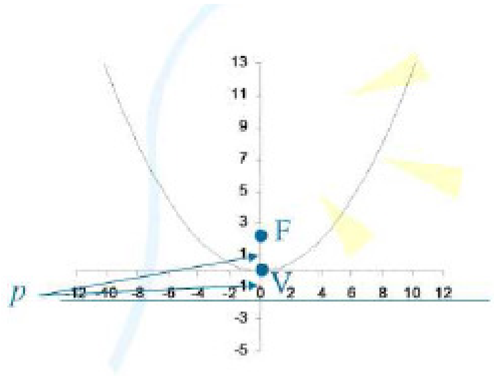
The <u>vertex and focus are points; the directrix and axis are lines.</u>

p is a very important value.

The **distance from the vertex to the focus and from the vertex to the directrix is *p* units**.



Horizontal Parabola



Vertical Parabola

If the p is positive, the parabola opens up or right; negative, opens down or left.

The distance from the focus to the directrix is $2p$ units.

If the parabola has a vertical axis and its vertex is at (0, 0), its formula is: $x^2 = 4py$
If the parabola has a horizontal axis and its vertex is at (0, 0), its formula is: $y^2 = 4px$

The proof of this is not too bad. I will do it for a vertical parabola:

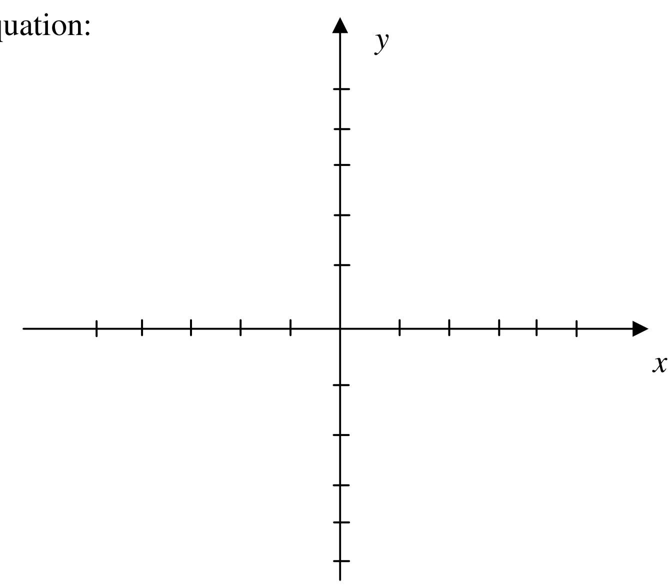
$$\sqrt{(x-0)^2 + (y-p)^2} = \sqrt{(x-x)^2 + (y+p)^2}$$
$$x^2 + (y-p)^2 = (y+p)^2$$
$$x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2$$
$$x^2 = 4py$$

The greater the distance between the focus and the vertex or greater the value of p, the 'fatter' or more wide the parabola.
The lesser the distance between the focus and the vertex or the lesser the value of p, the 'skinnier' or more narrow the parabola.

Find the following for each equation:

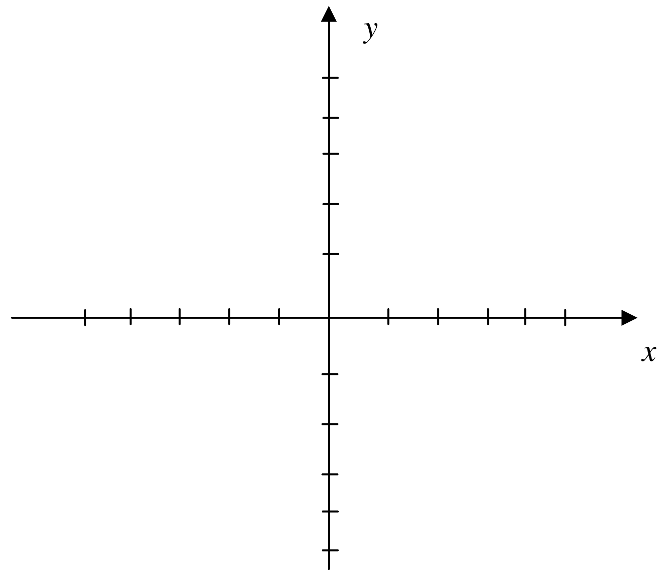
1. direction of opening
2. vertex
3. focus
4. directrix
5. sketch of graph

$x^2 = 8y$



Parabolas

$2y^2 = -5x$



If we take the standard equation of a parabola and replace *x* with *x* – *h* and *y* with *y* – *k*, then $x^2 = 4py$ becomes $(x-h)^2 = 4p(y-k)$ and $y^2 = 4px$ becomes $(y-k)^2 = 4p(x-h)$ with vertex V(*h*, *k*).

The following are standard forms of parabolas with a vertex at (*h, k*).

$$(x-h)^2 = 4p(y-k)$$

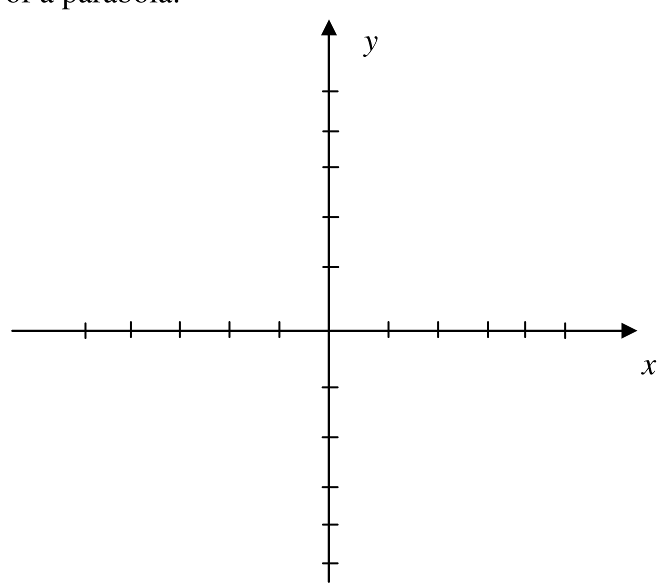
$$(y-k)^2 = 4p(x-h)$$

These are the formulas on the formula sheet.

Find the following for each equation of a parabola.

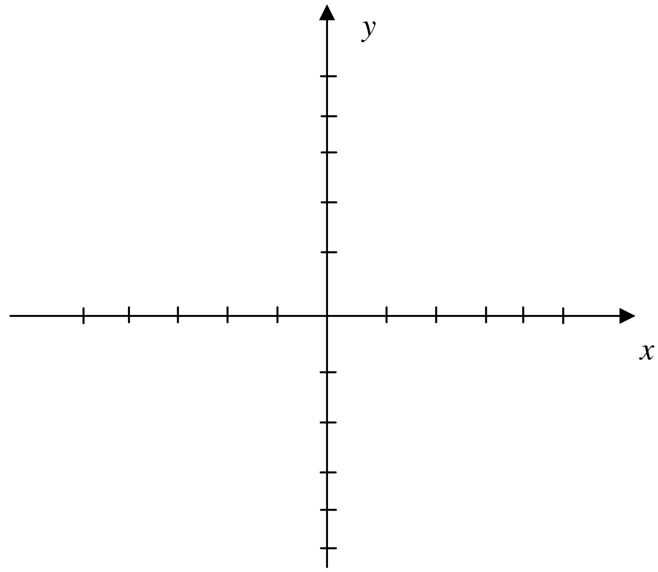
1. direction of opening
2. vertex
3. focus
4. directrix
5. sketch of graph

$(y-2)^2 = 12(x-1)$



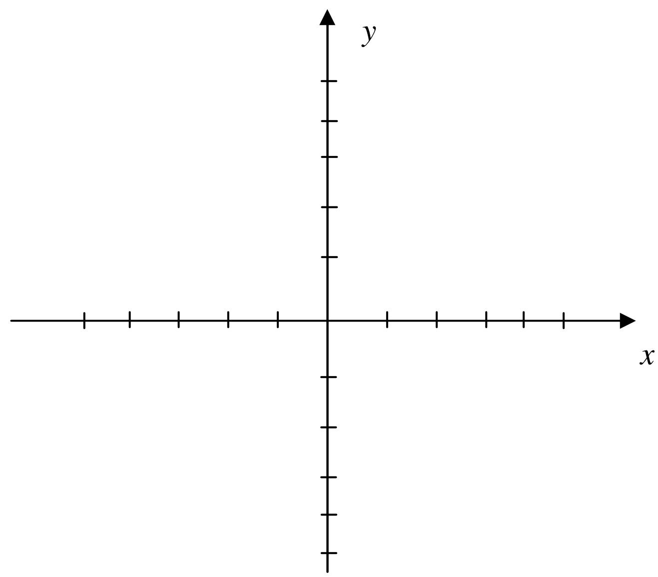
Parabolas

$(x+3)^2 = -\frac{1}{3}(y-2)$

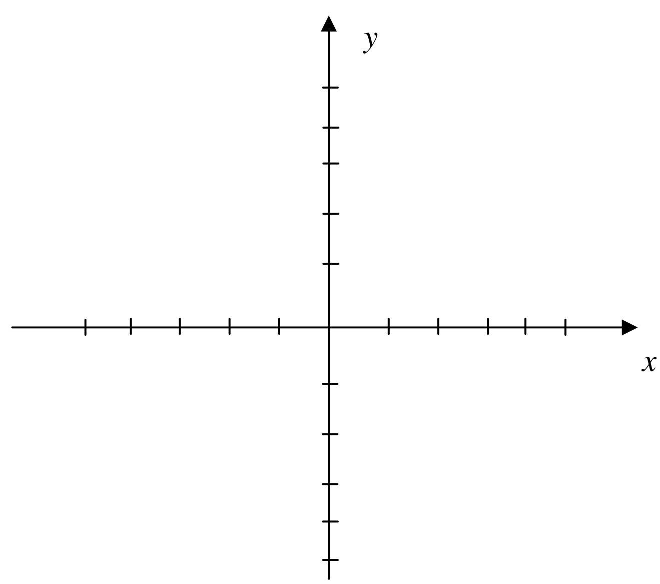


Sometimes completing the square will have to be used to get the equation in standard form.

$y^2 + 14y + 4x + 45 = 0$

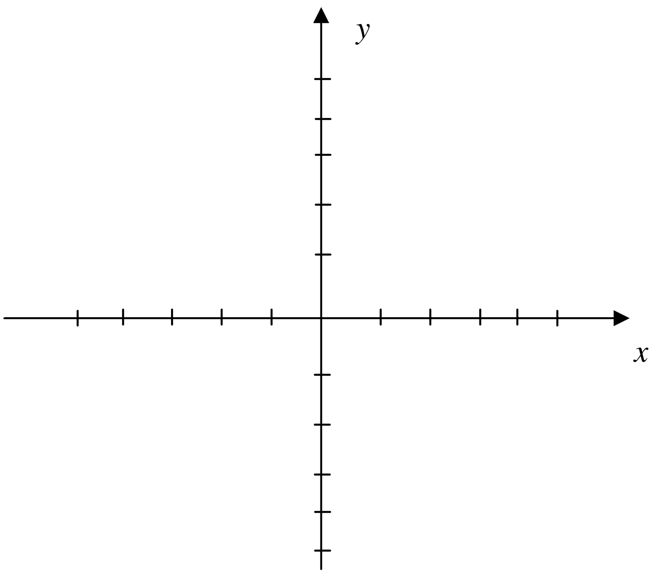


$x^2 + 20y = 10$



Sketch the parabola described and **find an equation** for the parabola. Hint: Make a rough sketch first to determine if the parabola is horizontal or vertical and to help find the value of p.

V(3, –1), F(3, 2)



V(–2, 3), F(–6, 3)