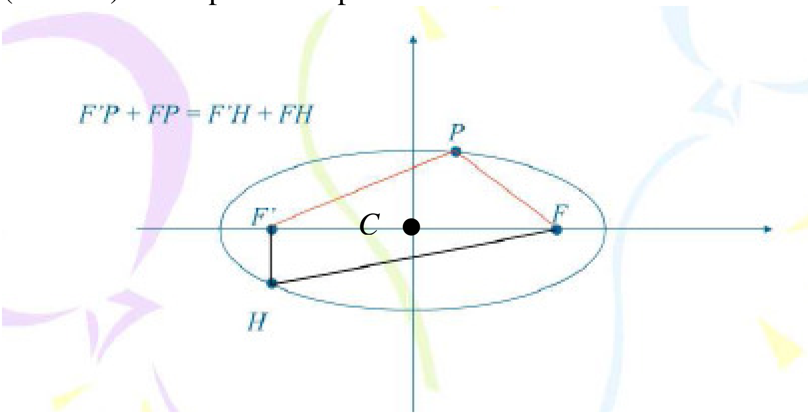
MA 15400 Lesson 27 Section 11.2

Ellipses

An **ellipse** is the set of all points in a plane, the sum of whose distances from two fixed points (the **foci**) in the plane is a positive constant.



Points F and F' are the foci (plural of focus). The sum of the distances from any point of an ellipse (such as *P* or *H*) to each focus is a constant value. For example, if

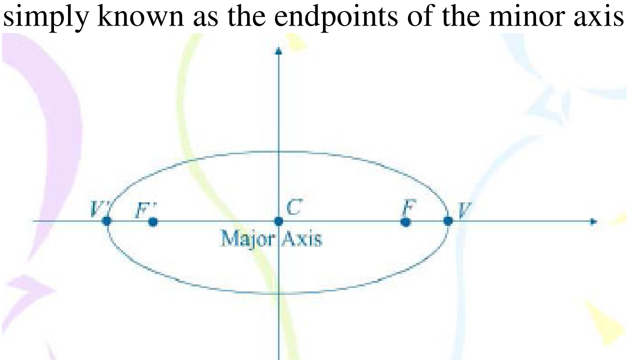
$FP + F'P = 10,$ then

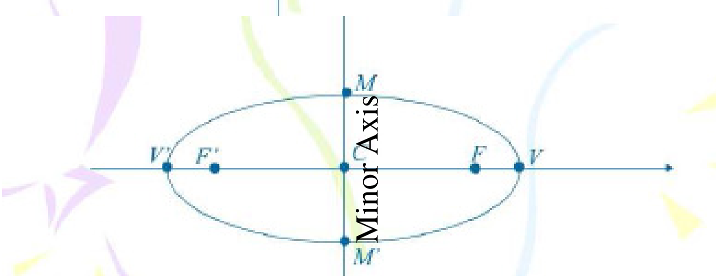
$FH + F'H = 10$

Notice the foci always lie 'inside' the ellipse.

The midpoint of the two foci is the **center** of the ellipse. The center *C* of the above ellipse is marked.

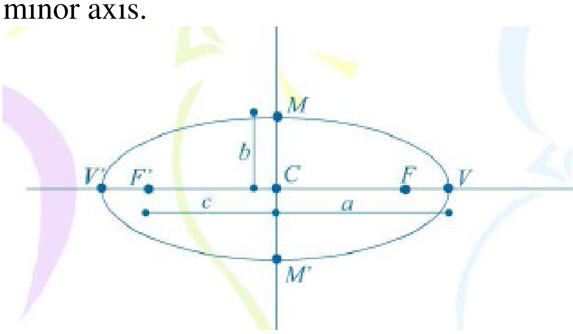
There are two axes: The **major axis** is the longer of the two and contains the foci. Its endpoints are the **vertices** of the ellipses. The **minor axis** is the shorter of the two and its endpoints are simply known as the endpoints of the minor axis.





The major axis of the ellipse at the left is the segment from V to V', which are called the vertices. <u>Vertices are always the endpoints of the major axis.</u> The minor axis (below left) is the segment form M to M'. In this ellipse the major axis is horizontal and the minor axis is vertical.

There are three important numbers associated with an ellipse: *a, b,* and *c*. These numbers correspond the distances involving the center, the foci, the vertices, and the endpoints of the minor axis.



The *a* value is the distance between the center a vertex. The *b* value is the distance between the center and an endpoint of the minor axis. The *c* value is the distance between the center and a focus.

Length of major axis = 2*a*

Length of minor axis = 2*b*

Length between 2 foci = 2*c*

Ellipses

Standard Equation of an Ellipse with **Center at the Origin**:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad or \quad \frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

Where $a>b>0$

Length of major axis = $2a$

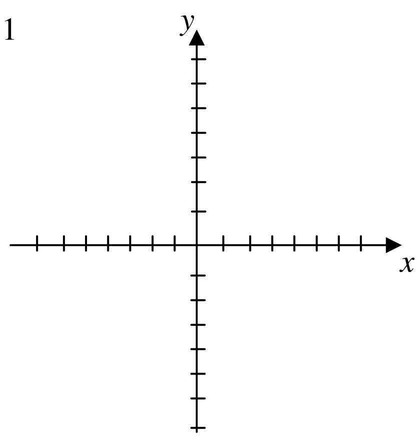
Length of minor axis = $2b$

Length between foci = $2c$ and $c^2=a^2-b^2$

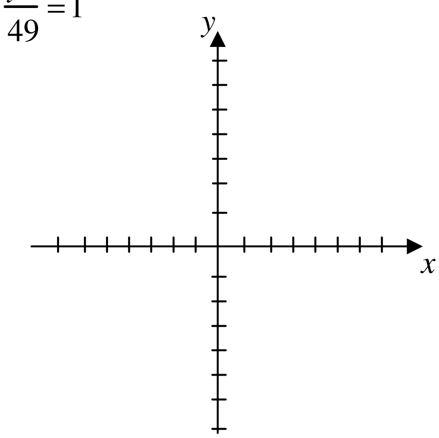
Notice, the a and b can be 'switched'. The a^2 always is the larger number and is associated with the major axis. If the ellipse has a horizontal major axis, the a^2 is under the x^2. If the ellipse has a vertical major axis, the a^2 is under the y^2.

Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.

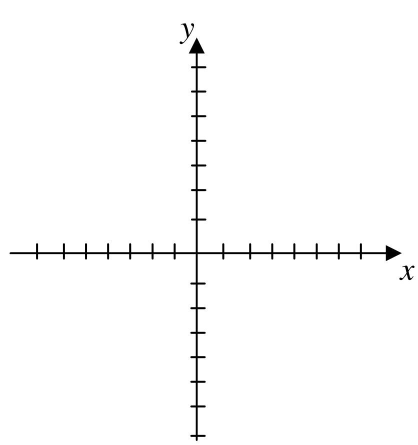
A $\frac{x^2}{36}+\frac{y^2}{9}=1$



B $\frac{x^2}{16}+\frac{y^2}{49}=1$



C $15x^2+9y^2=45$



When writing ordered pairs for vertices, foci, and endpoints of the minor axis; keep in mind whether to add/subtract the a, b, or c from the x or the y.

Ellipses

Standard Equation of an Ellipse with Center at (h, k):

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \quad or \quad \frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Where $a > b > 0$

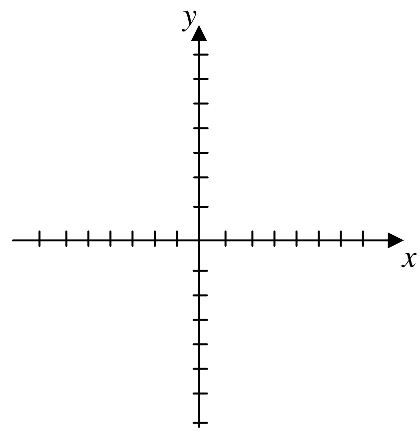
Length of major axis = $2a$

Length of minor axis = $2b$

Length between foci = $2c$ and $\boxed{c^2 = a^2 - b^2}$

Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.

D $\frac{(x-3)^2}{16}+\frac{(y+4)^2}{9}=1$



Center: $(3,-4)$

$a^2 = 16$ $b^2 = 9$ $c^2 = 16-9$

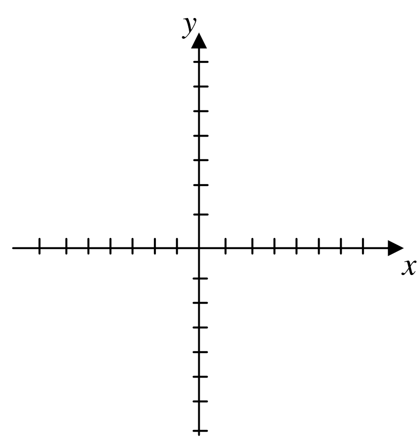
$a = 4$ $b = 3$ $c = \sqrt{7}$

Vertices: $(3 \pm 4, -4) \rightarrow$

Minor pts.: $(3, -4 \pm 3) \rightarrow$

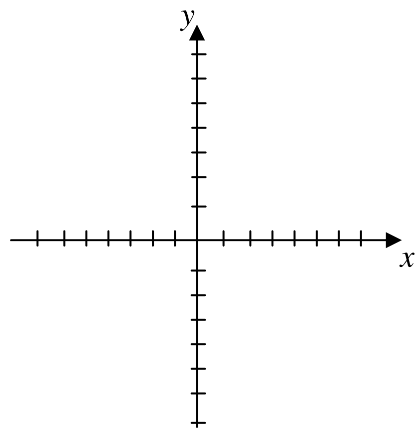
Foci: $(3 \pm \sqrt{7}, -4)$

E $2x^2 + 3y^2 + 8x - 24y - 4 = 0$ (A completing the square process will have to be used to get the equation in standard form.)

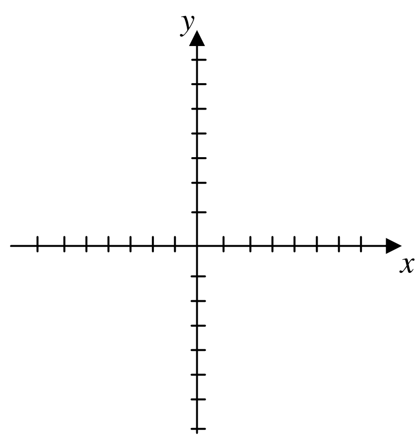


Ellipses

F $x^2 + 36y^2 + 6x - 72y + 9 = 0$



G $9(x-6)^2 + 32(y-1)^2 = 144$



Note: As the foci get closer to the center of the ellipse, the ellipse becomes more 'circular'. Then the foci converge on the center and becomes the center, the ellipse is a circle.

Ellipses

Find an equation for the ellipse with the following information.

H V(±4, 0), M(0,±3)

I V(0, ±5), M(± 2, 0)

J V(-1, 1) V' (-1, -3)
M(-2, -1) M'(0, -1)

K V(-1, 2) V'(9, 2)
M(4, 5) M'(4, -1)

(Sketches will help determine the equations.)