MA 15400 Lesson 3 Section 6.2

Trigonometric Function of Angles

Note: $(\sin\theta)^2$ is usually written $\sin^2\theta$. Similar notation for the other functions.)

The Fundamental Identities:

(1) The reciprocal identities:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

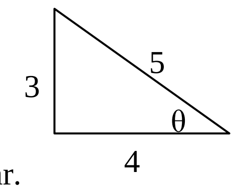
(2) The tangent and cotangent identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

(3) The Pythagorean identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

The reciprocal identities are obvious from the definitions of the six trigonometric functions.



Take the simple right triangle with sides 3, 4 and 5 with θ opposite the side of length 3.

To prove the tangent identity, examine the following. The cotangent identity proof is similar.

$$\tan\theta = \frac{opp}{adj} = \frac{opp}{hyp}\cdot\frac{hyp}{adj} = \frac{\frac{opp}{hyp}}{\frac{adj}{hyp}} = \frac{\sin\theta}{\cos\theta}$$

$$\sin\theta = \frac{3}{5} \qquad \cos\theta = \frac{4}{5} \qquad \tan\theta = \frac{3}{4}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{5}\cdot\frac{5}{4} = \frac{3}{4}$$

$$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}$$

Find sinθ and cosθ, now find $\sin^2\theta + \cos^2\theta$

$$\sin\theta = \frac{3}{5} \qquad \cos\theta = \frac{4}{5}$$

$$(\sin\theta)^2 + (\cos\theta)^2 = ?$$

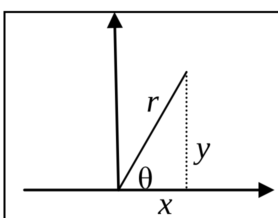
$$\sin^2\theta + \cos^2\theta = ?$$

$$\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = ?$$

$$\frac{9}{25} + \frac{16}{25} = ?$$

$$\frac{25}{25} = 1$$

$$\therefore \sin^2\theta + \cos^2\theta = 1$$



$$x^2 + y^2 = r^2 \qquad opp^2 + adj^2 = hyp^2$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2} \qquad \frac{opp^2}{hyp^2} + \frac{adj^2}{hyp^2} = \frac{hyp^2}{hyp^2}$$

$$\cos^2\theta + \sin^2\theta = 1 \qquad \sin^2\theta + \cos^2\theta = 1$$

$\sin^2\theta + \cos^2\theta = 1$ is a Pythagorean identity since it is derived from the Pythagorean Theorem.

Divide both sides by $\sin^2\theta$ to find another Pythagorean identity.

$$\sin^2\theta+\cos^2\theta=1 \text{ Divide each side by } \sin^2\theta$$
$$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$
$$1+\left(\frac{\cos\theta}{\sin\theta}\right)^2=\csc^2\theta$$
$$\therefore 1+\cot^2\theta=\csc^2\theta$$

The third Pythagorean identity can by found by dividing the original $\sin^2\theta+\cos^2\theta=1$ by $\cos^2\theta$.

Each of the three Pythagorean identities creates two more identities by subtracting a term from the left side to the right side.

$\sin^2\theta+\cos^2\theta=1$	$1+\tan^2\theta=\sec^2\theta$	$1+\cot^2\theta=\csc^2\theta$
$\sin^2\theta=1-\cos^2\theta$	$\tan^2\theta=\sec^2\theta-1$	$\cot^2\theta=\csc^2\theta-1$
$\cos^2\theta=1-\sin^2\theta$	$1=\sec^2\theta-\tan^2\theta$	$1=\csc^2\theta-\cot^2\theta$

Verify the identity by transforming the left side into the right side.

$\tan\theta\cot\theta=1$

$\sin(3\theta)\cot(3\theta)=\cos(3\theta)$

$$\frac{\sec\theta}{\tan\theta}=\csc\theta$$

$$\frac{\sin\left(\frac{\theta}{2}\right)+\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}=1+\cot\left(\frac{\theta}{2}\right)$$

Trigonometric Function of Angles

$(1-\cos\theta)(1+\cos\theta)=\sin^2\theta$

$\cos^2\theta\left(\sec^2\theta-1\right)=\sin^2\theta$

$\left(1-\sin^2\theta\right)\left(1+\tan^2\theta\right)=1$

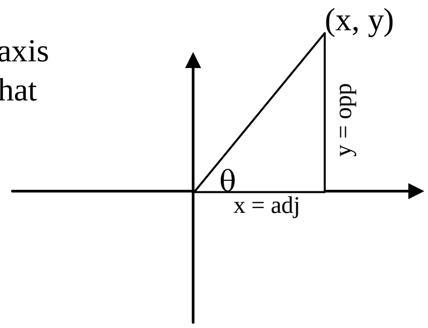
$\cot\theta+\tan\theta=\csc\theta\sec\theta$

Trigonometric Function of Angles

Using the coordinate system, draw an angle θ in **standard position** (vertex at the origin and the *x*-axis is the initial side).

Notice that the adjacent side corresponds to the x-value of the coordinate and the opposite side corresponds the y-value of the coordinate.

The idea that the cosine of θ corresponds to the x-axis and the sine of θ corresponds to the y-axis is one that you need to get used to. This is not saying that sin θ = the y value nor that cos θ = the x value. It simply says there is a correspondence.



If θ is an angle in **standard position** on a rectangular coordinate system and if P(-5, 12) is on the terminal side of θ, find the values of the six trigonometric functions of θ.

If θ is an angle in **standard position** on a rectangular coordinate system and if P(4, 3) is on the terminal side of θ, find the values of the six trigonometric functions of θ.

Trigonometric Function of Angles

Find the exact values of the six trigonometric functions of θ, if θ is in standard position and the terminal side of θ is in the specified quadrant and satisfies the given condition.

III; on the line $4x - 3y = 0$

II; parallel to the line $3x + y - 7 = 0$

Notice: tan θ = slope of the line!

Find the quadrant containing θ if the given conditions are true.

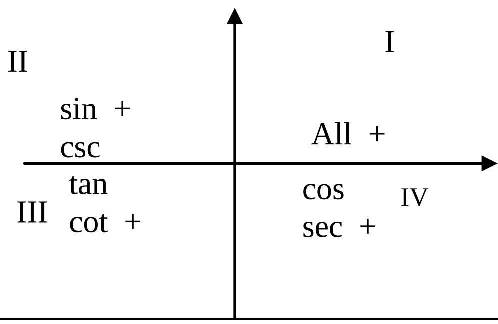
a) $\tan\theta < 0$ and $\cos\theta > 0$

b) $\sec\theta > 0$ and $\tan\theta < 0$

c) $\csc\theta > 0$ and $\cot\theta < 0$

d) $\cos\theta < 0$ and $\csc\theta < 0$

e) $\cos\theta < 0$ and $\sec\theta > 0$



To help you remember the picture above: Think (in order of quadrants): ALL STUDENTS TAKE CALCULUS.
Quadrant I: All functions are positive values.
Quadrant II: Sine and its inverse are positive, others negative.
Quadrant III: Tangent and its inverse are positive, others negative.
Quadrant IV: Cosine and its inverse are positive, other negative.

Trigonometric Function of Angles

Use the **fundamental identities** to find the values of the trigonometric functions for the given conditions:

$\tan\theta = \frac{12}{5}$ *and* $\cos\theta < 0$

$\sec\theta = -4$ *and* $\csc\theta > 0$