

**MA 16100 FINAL EXAM PRACTICE QUESTIONS**

1.  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 - x} =$  A. -1 B. 0 C. 1 D. 2 E. Does not exist
2. If  $y = (x^2 + 1) \tan x$ , then  $\frac{dy}{dx} =$  A.  $2x \tan x + (x^2 + 1) \sec^2 x$  B.  $2x \sec^2 x$   
C.  $2x \tan x + (x^2 + 1) \tan x$  D.  $2x \tan x + 2x \sec^2 x$  E.  $2x \tan x$
3. If  $h(x) = \begin{cases} x^2 + a, & \text{for } x < -1 \\ x^3 - 8 & \text{for } x \geq -1 \end{cases}$  determine all values of  $a$  so that  $h$  is continuous for all values of  $x$ . A.  $a = -1$  B.  $a = -8$  C.  $a = -9$  D.  $a = -10$  E. There are no values of  $a$ .
4. Evaluate  $\lim_{x \rightarrow 0^+} x \cos\left(\frac{1}{x}\right)$ . (Hint:  $-1 \leq \cos\left(\frac{1}{x}\right) \leq 1$  for all  $x \neq 0$ .) A. 0 B. 1  
C. -1 D.  $\frac{\pi}{2}$  E. Does not exist
5. If  $f(x) = \frac{1}{x+3}$ , then  $\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} =$  A.  $\frac{1}{4}$  B.  $\frac{1}{16}$  C.  $-\frac{1}{16}$  D.  $-\frac{1}{4}$  E. Does not exist
6. The equation  $x^3 - x - 5 = 0$  has one root in the interval  $(-2, 2)$ . This root is in the interval: A.  $(-2, -1)$  B.  $(-1, 0)$  C.  $(0, 1)$  D.  $(1, 2)$  E.  $(-1, 1)$
7. If  $f(x) = \frac{1-x}{1+x}$ , then  $f'(1) =$  A. -1 B.  $-\frac{1}{2}$  C. 0 D.  $\frac{1}{2}$  E. 1
8. If  $y = \ln(1-x^2) + \sin^2 x$ , then  $\frac{dy}{dx} =$  A.  $\frac{1}{1-x^2} + \cos^2 x$  B.  $\frac{1}{1-x^2} + 2 \sin x \cos x$   
C.  $\frac{1}{1-x^2} + 2 \sin x$  D.  $\frac{-2x}{1-x^2} + \cos^2 x$  E.  $\frac{-2x}{1-x^2} + 2 \sin x \cos x$
9. Find  $f''(x)$  if  $f(x) = \frac{1-x}{1+x}$  A.  $\frac{4}{(1+x)^3}$  B.  $-\frac{4}{(1+x)^3}$  C.  $-\frac{4x}{(1+x)^3} + \frac{2}{(1+x)^2}$   
D.  $\frac{2(1+x)^2 - 2x(1+x)}{(1+x)^4}$  E. -1
10. Assume that  $y$  is defined implicitly as a differentiable function of  $x$  by the equation  $xy^2 - x^2 + y + 5 = 0$ . Find  $\frac{dy}{dx}$  at  $(-2, 1)$ . A. 9 B.  $-\frac{5}{3}$  C. 1 D. 2 E.  $\frac{5}{3}$
11. Find the maximum and minimum values of the function  $f(x) = 3x^2 + 6x - 10$  on the interval  $-2 \leq x \leq 2$ . A. max is 14, min is -10. B. max is -10, min is -13 C. max is 14, min is -13 D. no max., min is -10 E. max is 14, no min.
12. For a differentiable function  $f(x)$  it is known that  $f(3) = 5$  and  $f'(3) = -2$ . Use a linear approximation to get the approximate value of  $f(3.02)$ . A. 6.02 B. 5.02 C. 5.04 D. 3 E. 4.96.
13. Water is withdrawn from a conical reservoir, 8 feet in diameter and 10 feet deep (vertex down) at the constant rate of  $5 \text{ ft}^3/\text{min}$ . How fast is the water level falling when the depth of the water in the reservoir is 5 ft? ( $V = \frac{1}{3}\pi r^2 h$ ). A.  $\frac{15}{16\pi} \text{ ft/min}$   
B.  $\sqrt{\frac{3}{\pi}} \text{ ft/min}$  C.  $\frac{2}{\pi} \text{ ft/min}$  D.  $5\sqrt[3]{3/4\pi} \text{ ft/min}$  E.  $\frac{5}{4\pi} \text{ ft/min}$ .

14. A rectangle is inscribed in the upper half of the circle  $x^2 + y^2 = a^2$  with its base on the x-axis. Calculate the area of the largest such rectangle.  
 A.  $\frac{a^2}{2}$  B.  $3a\sqrt{2}$  C.  $2a^2$  D.  $4a^2$  E.  $a^2$ .
15. Given that  $f(x)$  is differentiable for all  $x$ ,  $f(2) = 4$ , and  $f(7) = 10$ , then the Mean Value Theorem states that there is a number  $c$  such that  
 A.  $2 < c < 7$  and  $f'(c) = \frac{6}{5}$  B.  $2 < c < 7$  and  $f'(c) = \frac{5}{6}$  C.  $4 < c < 10$  and  $f'(c) = \frac{6}{5}$  D.  $2 < c < 7$  and  $f'(c) = 0$  E.  $4 < c < 10$  and  $f'(c) = 0$ .
16. Suppose that the mass of a radioactive substance decays from 18 gms to 2 gms in 2 days. How long will it take for 12 gms of this substance to decay to 4 gms?  
 A.  $\frac{\ln 3}{\ln 2}$  days B. 1 day C.  $\frac{\ln 2}{\ln 3}$  days D. 2 days E.  $(\ln 3)^2$  days
17. Which of the following is/are true about the function  $g(x) = 4x^3 - 3x^4$ ? (1)  $g$  is decreasing for  $x > 1$ . (2)  $g$  has a relative extreme value at  $(0, 0)$ . (3) the graph of  $g$  is concave up for all  $x < 0$ .  
 A. (1), (2) and (3) B. only (2) C. only (1) D. (1) and (2) E. (1) and (3).
18. Find where the function  $f(x) = 2/\sqrt{1+x^2}$  is increasing  
 A. all  $x$  B. no  $x$   
 C.  $x < 0$  D.  $x > 0$  E.  $x = 0$ .
19. Let  $f$  be a function whose derivative,  $f'$ , is given by  $f'(x) = (x-1)^2(x+2)(x-5)$ . The function has  
 A. a relative maximum at  $x = -2$  and a relative minimum at  $x = 5$ . B. a relative maximum at  $x = 5$  and a relative minimum at  $x = -2$ .  
 C. relative maxima at  $x = 1, x = -2$  and a relative minimum at  $x = 5$ . D. a relative maximum at  $x = 5$  and relative minima at  $x = 1, x = -2$  E. a relative maximum at  $x = 1$  and relative minima at  $x = -2, x = 5$ .
20. Find  $\frac{d}{dx} \int_1^{2x} \sqrt{t^2 + 1} dt$  at  $x = \sqrt{2}$ .  
 A. 6 B. 3 C.  $\sqrt{2}$  D.  $\sqrt{4x^2 + 1}$  E.  $\frac{1}{2\sqrt{3}}$ .
21.  $\int_3^4 x\sqrt{25-x^2} dx =$   
 A. 0 B. -37 C.  $\frac{37}{3}$  D.  $-\frac{74}{3}$  E.  $\frac{7}{12}$
22.  $\lim_{x \rightarrow \infty} \frac{x^2 + 2x}{3x^2 + 4} =$   
 A. 1 B.  $\frac{3}{7}$  C.  $\frac{1}{4}$  D. 0 E.  $\frac{1}{3}$ .
23.  $\lim_{x \rightarrow 0} \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x} =$   
 A.  $\frac{1}{2}$  B. 2 C.  $\frac{1}{3}$  D. 1 E. 0

24. For a certain function  $f(x)$ ,  $f''(x) = 6x$ ,  $f'(1) = 2$ , and  $f(2) = 3$ . On which intervals is  $f(x)$  increasing? A.  $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$  B.  $x < -\frac{1}{\sqrt{3}}$  and  $x > \frac{1}{\sqrt{3}}$  C. all  $x$  except  $x = 0$  D. all  $x$  E. no  $x$
25. Let  $R$  be the region between the graph of  $y = \frac{1}{x}$  and the  $x$ -axis, from  $x = a$  to  $x = b$  ( $0 < a < b$ ). If the vertical line  $x = c$  cuts  $R$  into two parts of equal area, then  $c =$   
A.  $\sqrt{ab}$  B.  $\frac{a+b}{2}$  C.  $\frac{\ln a + \ln b}{2}$  D.  $\ln\left(\frac{a+b}{2}\right)$  E.  $\ln\left(\frac{b-a}{2}\right)$
26. The area of the region between the graph of  $y = \frac{1}{1+x^2}$  and the  $x$ -axis, from  $x = -\sqrt{3}$  to  $x = 1$  is  
A.  $\frac{\pi}{2}$  B.  $\frac{3\pi}{4}$  C.  $\frac{5\pi}{12}$  D.  $\frac{\pi}{3}$  E.  $\frac{7\pi}{12}$
27.  $\frac{d}{dx}(e^{2x} \ln \sqrt{1+x}) =$   
A.  $e^{2x} \ln(1+x) + \frac{e^{2x}}{2(1+x)}$  B.  $\frac{e^{2x}}{\sqrt{1+x}} + 2e^{2x} \ln \sqrt{1+x}$   
C.  $\frac{1}{2}e^{2x} \ln(1+x) + \frac{e^{2x}}{2(1+x)}$  D.  $\frac{2e^{2x}}{\sqrt{1+x}}$  E.  $\frac{e^{2x}}{1+x}$
28.  $\frac{d}{dx}x^{\sin x} =$   
A.  $(\cos x)x^{\sin x}$  B.  $(\sin x)x^{\sin x-1}$  C.  $x^{\cos x}$  D.  $x^{\sin x}\left[\frac{\sin x}{x} + (\cos x) \ln x\right]$   
E.  $(\ln x)x^{\sin x}$
29.  $\frac{d}{dx} \tan^{-1} e^{3x} =$   
A.  $\frac{1}{1+e^{3x}}$  B.  $\frac{e^{3x}}{1+e^{3x}}$  C.  $\frac{3e^{3x}}{1+e^{6x}}$  D.  $\frac{3e^{3x}}{1+e^{9x^2}}$  E.  $\frac{3e^{3x}}{\sqrt{1-e^{6x}}}$
30.  $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} dx =$   
A.  $\frac{\pi}{2}$  B.  $\frac{\pi}{6}$  C.  $\sin^{-1} \sqrt{3}$  D.  $\frac{\pi}{3}$  E. 1
31.  $\int_0^4 \frac{x}{\sqrt{1+2x}} dx =$   
A.  $\frac{7}{2}$  B.  $\frac{10}{3}$  C.  $\frac{11}{4} \tan^{-1} 3$  D. 3 E. 4
32.  $\int_0^1 \frac{e^x}{1+e^x} dx =$   
A.  $\ln \frac{1+e}{2}$  B.  $\ln(1+e)$  C.  $\frac{1}{2}$  D.  $1 - \ln 2$  E.  $e$

33. If  $f(x) = \frac{4x-1}{2x+3}$ , then  $f^{-1}(x)$  is

- A.  $\frac{4+2x}{1-3x}$    B.  $\frac{1+3x}{4-2x}$    C.  $\frac{6+2x}{1+3x}$    D.  $\frac{4+2x}{1+3x}$    E.  $\frac{4-2x}{1-3x}$

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Answers: 1.D, 2.A, 3.D, 4.A, 5.C, 6.D, 7.B, 8.E, 9.A, 10.E, 11.C, 12.E, 13.E, 14.E, 15.A, 16.B, 17.C, 18.C, 19.A, 20.A, 21.C, 22.E, 23.E, 24.B, 25.A, 26.E, 27.A, 28.D, 29.C, 30.D, 31.B, 32.A, 33.B.