MATH 262 **PRACTICE PROBLEMS**

1. If $y' + (\tan x)y = \cos x \quad (0 < x < \pi/2)$, then $y =$

   A. $x\cos x + c$ B. $\cos^2 x - x + c$ C. $\ln(\cos^2 x + 1) + c$
   D. $x\cos x + c\cos x$ E. $\cot^2 x + \sec x + c$

2. The solution to the initial value problem
   $(\cos(x^2 + y) - 3xy^2)dy/dx + 2x\cos(x^2 + y) - y^3 = 0 \quad y(0) = 0$ is

   A. $\sin(x^2 + y) - xy^3 = 0$ B. $2x\cos(x^2 + y) - y^3 = 0$
   C. $x^2\cos(x^2 + y) - y^3 = 1$ D. $(2x + 1)\cos(x^2 + y) - y^3 = 1$
   E. $(2x + 1)\sin(x^2 + y) - y^3 = 0$

3. A tank initially holds a solution of 10 lb. of salt in 100 gallons of water. A salt solution containing 0.2 lb. of salt per gallons runs into the tank at the rate of 3 gallons per day. Water evaporates from the tank at the rate of 1 gallon per day and the well stirred solution runs out of the tank at the rate of 2 gallons per day. Let $x(t)$ be the amount of salt in the tank at time t (in days). Find a differential equation for $x(t)$.

   A. $x' = 0.4 - \frac{3x}{100}$ B. $x' = 0.1 - \frac{2x}{100}$ C. $x' = 0.6 - \frac{2x}{100-t}$
   D. $x' = 0.1 - 3x$ E. $x' = 0.6 - \frac{2x}{100}$

4. If $\dfrac{dy}{dx} = 3 - 4y$ and $y(0) = 0$, then $y\left(\dfrac{1}{2}\right) =$

   A. $\dfrac{2}{3}e^{-2}$ B. $\dfrac{3}{2}(1 - e^{-1})$ C. $\dfrac{3}{2}(1 - e^{-2})$ D. $\dfrac{3}{4}(1 - e^{-2})$ E. $3(1 - e^{-1})$

5. The values of α and β for which the system

$$\begin{aligned} x_1 - x_2 + x_3 &= \alpha \\ x_1 \qquad + 2x_3 &= \beta \\ 5x_1 - 2x_2 + 8x_3 &= 6 \end{aligned}$$

   has no solutions are

   A. $\alpha = 0,\ \beta = 2$ B. $\alpha = 3,\ \beta = 0$ C. $6 - 2\alpha - 3\beta \neq 0$
   D. $6 + 2\alpha - 5\beta = 0$ E. System always has a solution.

6. Let A be a nonsingular 3×3 matrix such that

$$A^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 3 \end{pmatrix}.$$

   The solution to the system $A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ is

   A. $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ B. $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ C. $\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$ D. $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ E. None of the above

7. Let A be the matrix

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 3 & 1 & 3 & 1 \\ 1 & 1 & 1 & 1 \\ 3 & 3 & 2 & 2 \end{bmatrix}$$

   The cofactor A_{23} of this matrix is

   A. 0 B. 1 C. -1 D. 2 E. -2

8. Exactly two of the following statements are true. Which are they?
(i) $\det(A+B)=\det(A)+\det(B)$ (ii) $\det(AB)=\det(A)\det(B)$
(iii) $\det(cA)=c\det(A)$ (iv) $\det(A^T)=\det(A)$

A. (i) and (ii) B. (i) and (iii) C. (ii) and (iii)
D. (ii) and (iv) E. (iii) and (iv)

9. Find the determinant of the matrix

$$\begin{bmatrix} 1 & 0 & 3 & 3 \\ 1 & 2 & 2 & 3 \\ 1 & 3 & 1 & 3 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

A. 0 B. 1 C. 4 D. −13 E. −17

10. A basis for the solution space of the system

$$\begin{aligned} x_1+2x_2+3x_3+\ x_4 &= 0 \\ 4x_1+5x_2+6x_3+4x_4 &= 0 \\ 7x_1+8x_2+9x_3+8x_4 &= 0 \end{aligned}$$

is

A. $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ B. $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ C. $\begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

D. $\begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 1 \\ 1 \end{pmatrix}$ E. $\begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}$

11. Which of the following sets of vectors are independent?
(a) $(0,0,1)(0,1,1),(0,3,2)$ (b) $(1,2,3),(4,5,6),(3,2,1)$
(c) $(0,0,0),(0,1,0),(1,0,0)$

A. a B. b C. c D. all E. none

12. If A is a 3×3 matrix and the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ are both solutions of the system of equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ then the system of equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has

A. two solutions B. no solution C. a unique solution
D. infinitely many solutions E. nothing can be determined from the information.

13. The subspace of $\mathbb{R}^3$ spanned by the vectors $(1,1,1),(1,1,0),(0,0,1),(-1,-1,-1)$ has dimension

A. 0 B. 1 C. 2 D. 3 E. 4

14. Which of the following sets of vectors span $\mathbb{R}^3$?
$S_1=\{(1,0,2),(0,2,4)\}$
$S_2=\{(1,2,0),(1,0,2),(1,1,1)\}$
$S_3=\{(1,2,0),(1,0,2),(1,1,1),(0,1,1)\}$

A. S_1,S_2 B. S_3 C. S_2,S_3 D. S_2 E. S_1,S_2,S_3

15. Let V be the solution space for the equation

$$\begin{bmatrix} 1 & -1 & 0 & 2 \\ 1 & -1 & 1 & 3 \\ 1 & -1 & -1 & 1 \end{bmatrix} X = 0$$

A basis for V is

A. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \right\}$ B. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \right\}$. C. $\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \right\}$. E. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

16. Determine all values of the constant k for which the vectors

$$(1,1,k,1), (-1,k,1,-1), (2,1,3,k)$$

are linearly independent

A. $k \neq -1$ B. All k C. $k = 1, k = 2$ D. $k \neq -1, k \neq 2$, E. No k

17. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x) = Ax$ where $A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & 2 & -3 \\ 0 & 0 & 0 \end{pmatrix}$. Then, a basis for Ker($T$) is

A. $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ B. $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$ C. $\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ D. $\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$ E. $\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

18. If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation such that $T(2x + 3y) = x - y$ and $T(x + 2y) = 2x + y$, then $T(x)$ is equal to:

A. $-4x - 5y$ B. $3x - 5y$ C. $-2x + 3y$ D. $x + y$ E. $5x$

19. Determine which of the following transformations $T : \mathbb{R}^3 \to \mathbb{R}^2$ are linear transformations.

i) $T((x_1, x_2, x_3)) = (x_1, x_1 - x_2)$
ii) $T((x_1, x_2, x_3)) = (x_1 - x_2, 1)$
iii) $T((x_1, x_2, x_3)) = (0, 0)$

A. i) B. i) and ii) C. i) and iii) D. ii) and iii) E. i), ii), and iii)

20. The inverse of the matrix $\begin{bmatrix} 2 & -1 \\ 8 & -5 \end{bmatrix}$ is

A. $\begin{pmatrix} 5 & -1 \\ 4 & -1 \end{pmatrix}$ B. $\begin{pmatrix} 5/2 & 1 \\ 4 & 2 \end{pmatrix}$ C. $\begin{pmatrix} 5/2 & -1/2 \\ 4 & 1 \end{pmatrix}$ D. $\begin{pmatrix} 5/2 & -1/2 \\ 4 & -1 \end{pmatrix}$ E. Matrix has no inverse

21. The eigenvalues of $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ are

A. $2, -2$ B. $3, -3$ C. $-4, 2$ D. $1, 1$ E. $4, -2$

22. The eigenvalues of $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ are 0 and 2. An eigenvector corresponding to 2 is

A. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ B. $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ C. $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ D. $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ E. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

23. The eigenvalues of the matrix $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ are 2, 0, and -1. An eigenvector corresponding to -1 is

A. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ B. $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ C. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ D. $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ E. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

24. The general solution of $y'' - 3y' - 4y = 0$ is $y = c_1 e^{-x} + c_2 e^{4x}$. Which form would you use to determine a particular solution to the equation $y'' - 3y' - 4y = xe^{-x} + \cos 2x$ by undetermined coefficients?

A. $Axe^{-x} + B\cos 2x$ B. $Axe^{-x} + B\cos 2x + C\sin 2x$ C. $x(A + Bx)e^{-x} + C\cos 2x$
D. $x(A + Bx)e^{-x} + C\cos 2x + D\sin 2x$ E. $Ax^2e^{-x} + B\cos 2x + C\sin 2x$

25. One solution of the differential equation $x^3y'' + 2xy' - 2y = 0$, $x > 0$ is $y_1 = x$. Another solution is of the form $y_2 = vx$ where v satisfies the differential equation

A. $v'' + xv' = 0$ B. $x^2v'' + 2xv' = 0$ C. $x^2v'' + (x+1)v' = 0$ D. $(x+1)v'' + x^2v' = 0$
E. $x^2v'' + 2(x+1)v' = 0$

26. An annihilator of $\cos x + xe^x + 1$ is:

A. $D(D-1)(D+1)^2$ B. $(D^2-1)^2(D+1)^2$ C. $D(D-1)(D^2+1)^2$
D. $D(D-1)^2(D^2+1)$ E. $D(D-1)(D^2+1)$

27. The general solution of $y'''' - y''' = 0$ is $y =$

A. $c_1e^x + c_2$ B. $c_1e^x + c_2x + c_3$ C. $c_1e^x + c_2x^2 + c_3x + c_4$
D. $c_1e^x + c_2x^3 + c_3x^2 + c_4x + c_5$ E. $c_1e^x + c_2(1 + x + x^2)$

28. The constants a, b, c are real and the roots of the equation $ar^2 + br + c = 0$ are $r = 1 \pm 2i$. The general solution of the differential equation $ay'' + by' + cy = 0$ is $y =$

A. $e^x(c_1 \cos 2x + c_2 \sin 2x)$ B. $e^{2x}(c_1 \sin x + c_2 \cos x)$ C. $c_1e^x + c_2e^{2x}$
D. $c_1 \cos x + c_2 \sin x$ E. $c_1 \cos 2x + c_2 \sin 2x$

29. The solution to $y'' - y = e^x$ satisfying $y(0) = 1$, $y'(0) = 3/2$ satisfies $y(1) =$

A. $\frac{3}{2}e$ B. $\frac{7}{4}e - \frac{1}{4}e^{-1}$ C. $\frac{1}{2}e$ D. $\frac{1}{2}e + e^{-1}$ E. $\frac{5}{4}e - \frac{1}{4}e^{-1}$

30. If $A = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$ and if one solution of the equation $x' = Ax$ is $x = \begin{bmatrix} 0 \\ e^{2t} \end{bmatrix}$, then another linearly independent solution is

A. $\begin{bmatrix} e^{2t} \\ e^{3t} \end{bmatrix}$ B. $\begin{bmatrix} 0 \\ e^{2t}+e^{3t} \end{bmatrix}$ C. $\begin{bmatrix} e^{3t} \\ 0 \end{bmatrix}$ D. $\begin{bmatrix} e^{3t} \\ e^{2t} \end{bmatrix}$ E. $\begin{bmatrix} e^{3t} \\ e^{2t}+e^{3t} \end{bmatrix}$

31. If a fundamental matrix for the system $x' = Ax$ is $X(t) = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^t \end{bmatrix}$, find a particular solution, $x_p(t)$ of $x' = Ax + \begin{bmatrix} 2 \\ e^t \end{bmatrix}$, such that $x_p(0) = 0$.

A. $\begin{bmatrix} 2e^t-2 \\ t \end{bmatrix}$ B. $\begin{bmatrix} 2e^t \\ t \end{bmatrix}$ C. $\begin{bmatrix} 2-2e^{-t} \\ te^t \end{bmatrix}$ D. $\begin{bmatrix} te^{-t} \\ 2-2e^t \end{bmatrix}$ E. $\begin{bmatrix} te^t \\ 2 \end{bmatrix}$

32. The general solution to the linear system

$$\frac{dx}{dt} = Ax, \quad A = \begin{bmatrix} 0 & -2 \\ 1 & 2 \end{bmatrix}$$

has the form

A. $c_1e^t \begin{bmatrix} \cos t+\sin t \\ -\cos t \end{bmatrix} + c_2e^t \begin{bmatrix} \cos t-\sin t \\ \sin t \end{bmatrix}$ B. $c_1e^{-t} \begin{bmatrix} \cos t+\sin t \\ -\cos t \end{bmatrix} + c_2e^{-t} \begin{bmatrix} \cos t-\sin t \\ \sin t \end{bmatrix}$
C. $c_1e^t \begin{bmatrix} \cos t-\sin t \\ \cos t \end{bmatrix} + c_2e^t \begin{bmatrix} 2\cos t \\ \sin t \end{bmatrix}$ D. $c_1e^t \begin{bmatrix} \cos 2t \\ -\cos 2t+\sin 2t \end{bmatrix} + c_2e^t \begin{bmatrix} \cos 2t-\sin 2t \\ \sin 2t \end{bmatrix}$
E. None of the above

Answers: 1.D, 2.A, 3.E, 4.D, 5.C, 6.C, 7.C, 8.D, 9.B, 10.E, 11.E, 12.D, 13.C, 14.B, 15.C, 16.D, 17.D, 18.A, 19.C, 20.D, 21.E, 22.A, 23.C, 24.D, 25.E, 26.D, 27.C, 28.A, 29.A, 30.E, 31.C, 32.A