1.1

In the set of integers a:

positive + positive $\Rightarrow$ _______ pos (pos) $\Rightarrow$ ________

negative + negative $\Rightarrow$ ______ neg(neg) $\Rightarrow$ _________

positive + negative $\Rightarrow$
negative + positive $\Rightarrow$ ___________ pos(neg) $\Rightarrow$
neg(pos) $\Rightarrow$ ________

EX: If x < 0 and y > 0, determine the sign of the real number:

a) xy b) $x^2 y$ c) $\frac{x}{y} + x$ d) y – x

EX: replace the line with <, >, or =:

a) -7 ______ -4 b) $\frac{\pi}{2}$ ______ 1.57 c) $\sqrt{225}$ ______15

d) -3 ______ -5 e) $\frac{\pi}{4}$ ________.8 f) $\sqrt{289}$ ______17

EX: Write the expressions using symbols:

a) x is negative ____________________

b) x is less than or equal to π ________________

c) d is between 4 and 2 _____________________

d) the reciprocal of x is at least 9: __________________

Absolute Value:____

To solve an expression involving absolute value:

1) Answer the question inside the absolute value sign
2) do the "outside operations

EX: Simplify:

a) $|-11+1|$ b) $|6|-|-3|$ c) $|8|+|-9|$ d) $(-5)|3-6|$

e) $\frac{|-6|}{-2}$ f) $|7|+|6-12|$

1.2

EXPONENTIAL NOTATION/ RULES/ ROOTS:

- $a^n = a \cdot a \cdot a \cdot ... \cdot a$ (ex: $a^4 = a \cdot a \cdot a \cdot a$)
- $ca^n = c \cdot a \cdot a \cdot .... \cdot a$ (ex: $5x^2 = 5 \cdot x \cdot x$)
- $a^0 = 1$
- $a^{-n} = \dfrac{1}{a^n}$
- $a^m a^n = a^{m+n}$
- $\left(a^m\right)^n = a^{mn}$
- $\left(ab\right)^n = a^n b^n$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
- $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n = \dfrac{b^n}{a^n}$
- $\dfrac{a^{-m}}{b^{-n}} = \dfrac{b^n}{a^m}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $\sqrt[r]{a^p} = a^{\frac{p}{r}}$ (may be written as: $\left(\sqrt[r]{a}\right)^p$)
- $\sqrt[r]{ab} = \sqrt[r]{a} \cdot \sqrt[r]{b}$
- $\sqrt[r]{\dfrac{a}{b}} = \dfrac{\sqrt[r]{a}}{\sqrt[r]{b}}$

EX:

1: Express the number in the form $\frac{a}{b}$ where a and b are integers (in other words: simplify)

a) $\left(\frac{-2}{3}\right)^4 \Rightarrow$

b) $(-3)^3 \Rightarrow$

c) $\frac{4^{-2}}{3^{-3}} \Rightarrow$

d) $8^{\frac{2}{3}} \Rightarrow$

e) $8^{\frac{-2}{3}} \Rightarrow$

2: Simplify: ***houses***

a) $\left(\frac{1}{2}x^4\right)\left(16x^5\right) \Rightarrow$

b) $\left(-4b^3\right)\left(\frac{1}{6}b^2\right)\left(-9b^4\right) \Rightarrow$

c) $\left(-5x^{-3}\right)\left(2x^5\right) \Rightarrow$

d) $\frac{\left(6x^3\right)^2}{\left(2x^2\right)^3} \Rightarrow$

e) $\left(3x^3\right)^4\left(4y^2\right)^{-3} \Rightarrow$

f) $\left(25z^4\right)^{\frac{-3}{2}} \Rightarrow$

g) $\left(\dfrac{4x^7y^3}{x^0y^{-5}}\right)^2 \Rightarrow$

h) $(27x)^{\frac{1}{3}}\left(3x^{\frac{1}{2}}\right) \Rightarrow$

i) $\left(\dfrac{-36x^4}{y^{-8}}\right)^{\frac{1}{2}} \Rightarrow$

3: Rewrite using rational exponents:

a) $\sqrt{x^4+y^4} \Rightarrow$ b) $\sqrt[4]{x^3} \Rightarrow$ c) $\sqrt[3]{(a+b)^2} \Rightarrow$

4: Rewrite using radicals:

a) $4x^{\frac{3}{2}} \Rightarrow$ b) $(4x)^{\frac{3}{2}} \Rightarrow$

c) $8-y^{\frac{1}{3}} \Rightarrow$ d) $(8-y)^{\frac{1}{3}} \Rightarrow$

5: Simplify: ***hedge***

a) $\sqrt{81} \Rightarrow$ b) $\sqrt[3]{648} \Rightarrow$ c) $\sqrt{200} \Rightarrow$

d) $\sqrt[5]{-64} \Rightarrow$ e) $\sqrt{9x^{-4}y^6} \Rightarrow$ f) $\sqrt{6x^2y^5} \cdot \sqrt{8xy^4} \Rightarrow$

6: Simplify and rationalize the denominator, if needed:

a) $\sqrt{\dfrac{1}{3}}$ b) $\sqrt{\dfrac{1}{3x^3y}}$ c) $\dfrac{4x}{\sqrt{2x^2y}}$

1.3

- monomial:________________________
- binomial:_________________________
- trinomial:_________________________
- polynomial:_______________________

Adding and subtracting polynomial: Combine like terms (like terms have the EXACT same variable part)
*****Clean up****

EX: Express as a polynomial:

a) $\left(3x^3+4x^2-7x+1\right)+\left(9x^3-4x^2-6x\right)$

b) $\left(6x^3-2x^2+x-2\right)-\left(8x^2-x+2\right)$

Multiplying and dividing polynomials: distribute each term of the 1st polynomial to each term of the 2nd polynomial, then combine like terms.

EX: Express as a polynomial:

a) $(2x+5)(3x-7)\Rightarrow$

b) $(3x+5)\left(2x^2+9x-5\right)\Rightarrow$

c) $(3x+2y)(3x-2y)\Rightarrow$

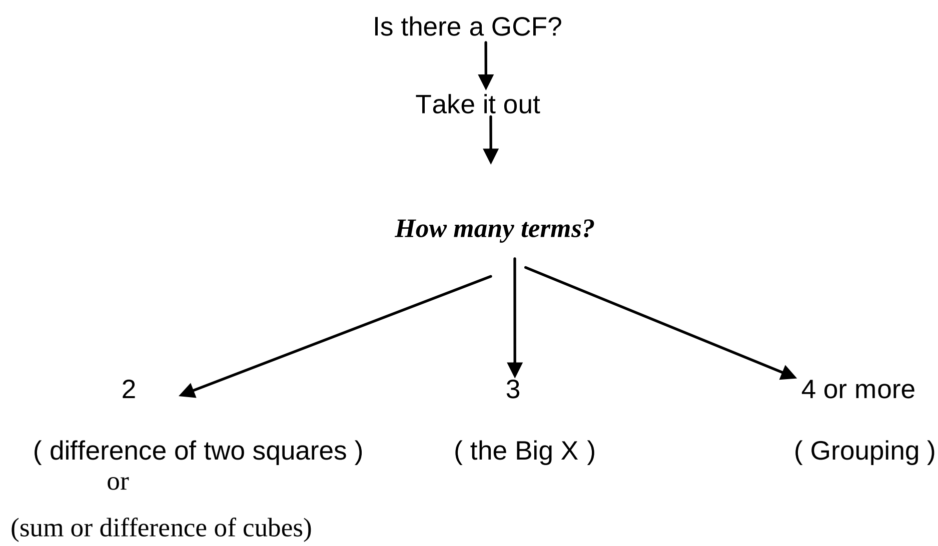
d) $\left(\sqrt{x}-\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)\Rightarrow$

e) $(x-2y)^3\Rightarrow$

Dividing a polynomial by a monomial: Separate into individual fractions, simplify:

EX: simplify: $\dfrac{8x^2y^3 - 10x^3y + 12x^4y^5}{2x^2y} \Rightarrow$

FACTORING POLYNOMIALS: Follow this chart:



Greatest common factor: Every term has a common factor: take it out and un-distribute

EX: Factor: $4x^2y + 8y^2 \Rightarrow$ the GCF is 4y

$\Rightarrow 4y(x^2 + 2y)$

Difference of 2 perfect squares: There are exactly 2 terms and both of them are perfect squares. It must be a SUBTRACTION problem. This factors into what I call "something plus, something minus"

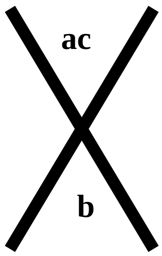
EX: Factor: $25x^2 - 16 \Rightarrow (5x+4)(5x-4)$

Cubes: There are exactly 2 factors and both are perfect cubes. It may be either addition or subtraction. This factors into what I call " little parenthesis, big parenthesis"

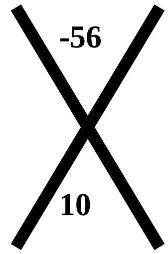
EX: Factor: $x^3 \pm y^3 \Rightarrow (x \pm y)(x^2 \quad xy + y^2)$

(what did you cube...same sign...what did you cube)(1st term squared ...opp sign...1st term x 2nd term...+ 2nd term squared)

The big X: There are 3 terms in the order $ax^2 + bx + c$. You make a big x....Then find the values you need that will multiply together to equal your top number and add together to equal you bottom number.



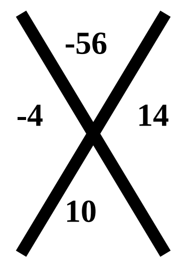
Ex: Factor: $7x^2 + 10x - 8$



Now look for the factors of -56 that will add up to 10: your choices are

-1(56) adds up to 55 --
-2(28) " 26
-4(14) " 10

So the factors you're going to use are -4 and 14. Put these into the big x:



Then re-write the original equation substituting these values in for the middle term: so you'll write -4x + 14 x instead of 10x

$$7x^2 + -4x + 14x - 8$$

From here you'll use **grouping:**

For **grouping**, you group the first two terms and the last two terms:

$$\left(7x^2 + -4x\right) + \left(14x - 8\right)$$

Then take out the GCFof each group:

$$x\left(7x + -4\right) + 2\left(7x + -4\right)$$

Then group together the "like" parenthesis and the "leftovers":

$$\left(7x - 4\right)\left(x + 2\right)$$

This is your final factored answer.

EX: Factor completely:

a) $rs + 4st \Rightarrow$

b) $x^2 - 49 \Rightarrow$

c) $x^4 - 4x^2 \Rightarrow$

d) $x^3 - 81x \Rightarrow$

e) $x^{16} - 1 \Rightarrow$

f) $x^2 + 3x + 4 \Rightarrow$

g) $6x^2 + 7x - 20 \Rightarrow$

h) $4x^2 - 20x + 25 \Rightarrow$

i) $4x^3+4x^2+x \Rightarrow$

j) $7x^2+10x-8 \Rightarrow$

k) $x^3+y^3 \Rightarrow$

l) $8x^3-125y^3 \Rightarrow$

m) $10x^2+11xy-6y^2 \Rightarrow$

1.4

Fractional expressions: This requires factoring:-)

To multiply or divide:
1) factor every polynomial
2) Cancel common factors
3) Set restrictions

EX: Simplify:

a) $\dfrac{2x^2+9x-5}{3x^2+17x+10} \Rightarrow$

b) $\dfrac{4x^2-9}{2x^2+7x+6} \bullet \dfrac{4x^2-4x-15}{x^2-2x-8} \Rightarrow$

c) **(when dividing fractions, remember to invert and multiply)**

$$\frac{3x^2+8x+4}{x^4-81} \div \frac{x^2-4}{x^3+6x^2+9x} \Rightarrow$$

Adding and subtracting fractional expressions:

1) Factor the denominator and find the LCD
2) multiply the numerators by the missing factors to find equivalent fractions
3) Combine like terms in the numerator
4) Factor the new numerator
5) Cancel, if possible

EX: Simplify:

a) $$\frac{15}{x^2-9} - \frac{5x}{x^2-9} \Rightarrow$$

b) $$\frac{4}{(5x-2)^2} + \frac{x}{5x-2} \Rightarrow$$

c) $$\frac{3x}{x+2} + \frac{5x}{x-2} - \frac{40}{x^2-4} \Rightarrow$$

d) $\frac{12x}{2x+1}-\frac{3}{2x^2+x}+\frac{5}{x} \Rightarrow$

When they give you the problem as a complex fraction, just separate it into top and bottom and then set it up as a division problem:

EX: Simplify: $\dfrac{\frac{1}{x+2}-3}{\frac{4}{x}-x} \Rightarrow$

EX: $\frac{y^{-2}-x^{-2}}{y^{-2}+x^{-2}} \Rightarrow$

Rationalize the denominator: You cannot leave a radical in the denominator of a fraction – to get rid of the radical you multiply by the conjugate of the denominator (the same terms with the opposite sign)

EX: $\frac{5}{\sqrt{3}+x}$ $\frac{a+b}{\sqrt{a}+\sqrt{b}}$ $\frac{3}{\sqrt[3]{5}}$

2.1 Equations: (These will be "solve" problems...they'll have an " =")

Linear equations: there will be no exponent greater than 1. Your mission is to get:

all variable terms = all constant terms (no variables) and then divide by the coefficient of the variable term. You need to find out what the variable equals.

** You need to remember to check for extraneous solutions if you have a variable in the denominator!!

EX: Solve: 4x – 7x + 2 = 3x – 3 – 5

To mess with your mind they will give you the equation as rational expressions: If there is only 1 fraction on each side, cross multiply to get it into linear form:

EX: $\dfrac{3+5x}{5}=\dfrac{4-x}{7}$

EX: $\dfrac{3x+1}{6x-2}=\dfrac{2x+5}{4x-13}$

If there are 2 or more fractions on a side, you have to find the common denominator and then multiply EACH fraction by that denominator to eliminate it:

EX: $\dfrac{4}{2x-3}+\dfrac{10}{4x^2-9}=\dfrac{1}{2x+3}$

SPECIAL CASES: If the variable disappears and all you have left are numbers, you need to look at the result and see if you have a true or false statement. If it is true, your answer is "all real numbers" (check for extraneous solutions). If it is false, your answer is "no solution"

EX: Solve: $\dfrac{1}{2x-1}=\dfrac{4}{8x-4}$

EX: $3x+7-2=3(x-6)$

EX: Solve: $\dfrac{-3}{x+4}+\dfrac{7}{x-4}=\dfrac{-5x+4}{x^2-16}$

If they give you a solution, substitute that value in for "x" and then solve for the other variable:

EX: 3x – 2 + 6c = 2c – 5x + 1: x = 4

To solve for 1 variable in terms of another, manipulate the expression to get the needed variable alone: Hints: If there is any distribution to do, do it first. 2nd: Move terms that are being added to the term with the needed variable. 3rd: Divide by the coefficient of the needed variable.

EX: Solve for b: $A=\frac{1}{2}(a+b)h$ Solve for x: $5x+7b=3$

2.2 Applied problems ...THOSE WONDERFUL STORY PROBLEMS!!!

Formulas: 1st: identify the variable 2nd: You will FSS (formula, substitute, solve)

We are just going to set these up at this time – if there is time, we will go back and solve them all.

1) A student in an algebra course has test scores of 75, 82, 71, and 84. What score on the next test will raise the student's average to 80?

2) A couple does not wish to spend more than $70 for dinner at a restaurant. If a sales tax of 6% is added to the bill and they plan to tip 15% after the tax has been added, what is the most they can spend for the meal?

3) A workman's basic hourly wage is $10, but he receives one and a half times his hourly rate for any hours worked in excess of 40 per week. If his paycheck for the week is $595, how many hours of overtime did he work?

4) An algebra student has won $100,000 in the lottery and wants to deposit it in savings accounts in two different institutions. One account pays 8% simple interest, but deposits are only insured up to $50,000. The second account pays 6.4% simple interest, and deposits are insured up to $100,000. Determine whether the money can be deposited so that it is fully insured and earns an annual interest of $7500.

5) A city government has approved the construction of an $800 million sports arena. Up to $480 million will be raised by selling bonds that pay simple interest at a rate of 6% annually. The remaining amount (up to $640 million) will be obtained by borrowing money form an insurance company at a simple interest rate of 5%. Determine whether the arena can be financed so that the annual interest is $42 million.

6) Six hundred people attended the premiere of a motion picture. Adult tickets cost $9, and children were admitted for $6. If box office receipts totaled $4800, how many children attended the premiere?

7) A consulting engineer's time is billed at $60 per hour, and her assistant's is billed at $20 per hour. A customer received a bill for $580 for a certain job. If the assistant worked 5 hours less than the engineer, how much dime did each bill on the job?

8) A pharmacist is to prepare 15 mL of special eye drops for a glaucoma patient. The eye-drop solution must have a 2% active ingredient, but the pharmacist only has 10% solution and 1% solution in stock. How much of each type of solution should be used to fill the prescription?

9) Theophylline, an asthma medicine, is to be prepared from an elixir with a drug concentration of 5 mg/mL and a cherry-flavored syrup that is to be added to hide the taste of the drug. How much of each must be used to prepare 100 mL of solution with a drug concentration of 2 mg/mL?

10) Two children, who are 224 meters apart, start walking toward each other at the same instant at rates of 1.5 m/sec and 2 m/sec respectively. When will they meet AND how far will each have walked?

11) At 6 AM a snowplow, traveling at a constant speed, begins to clear a highway leading out of town. AT 8 AM a car begins traveling the highway at a speed of 30 mph and reaches the plow 30 minutes later. Find the speed of the snowplow.

12) Two children own two-way radios that have a maximum range of 2 miles. One leaves a certain point at 1:00 PM walking due north at a rate of 4 mph. The other leaves the same point at 1:15 PM traveling due south at 6 mph. When will they be unable to communicate with one another?

13) A boy can row a boat at a constant rate of 5 mi/hr in still water. He rows upstream for 15 minutes and then rows downstream, returning to his starting point in another 12 minutes. Find the rate of the current AND the total distance he traveled.

14) A farmer plans to use 180 feet of fencing to enclose a rectangular region, using part of a straight river bank instead of fencing as one side of the rectangle. Find the area of the region if the length of the side parallel to the river bank is:

a) twice the length of an adjacent side b) ½ the length of an adjacent side c) the same length of an adjacent side

15) In looking at the cross section of a design for a two-story home, the center height, *h*, of the second story has not yet been determined. Find *h* such that the second story will have the same cross-sectional area as the first story.

16) A stained glass window is being designed in the shape of a rectangle surmounted by a semicircle. The width of the window is to be 3 feet, but the height is yet to be determined. If 24 square feet of glass is to be used, find the height of the window, h.

17) A large grain silo is to be constructed in the shape of a circular cylinder with a hemisphere attached to the top. The diameter of the silo is to be 30 ft, but the height is yet to be determined. Find the height, *h*, of the silo that will result in a capacity of 11,250 π cubic feet.

18) It takes a boy 90 minutes to mow the lawn, but his sister can mow it in 60 minutes. How long would it take them to mow the lawn if they worked together using 2 lawn mowers?

19) With water from one hose, a swimming pool can be filled in 8 hours. A second, larger hose used alone can gill the pool in 5 hours. How long would it take to fill the pool if both hoses were used simultaneously?

20) It takes a girl 45 minutes to deliver papers on her route; however, if hr brother helps, it takes them only 20 minutes. How long would it take her brother to deliver the papers by himself?

2.3 Quadratic equations: Must be in the form: $ax^2 + bx + c = 0$

Special Cases: If you have 1 variable term and 1 constant term, you can take the square root of both sides to find the solution. Remember, when you take the square root, you have to include BOTH roots, the positive and the negative.

EX:

1)Solve: $x^2 = 81$ 2)Solve: $(x-3)^2 = 18$ 3)solve: $121x^2 = 169$

True or false: If $x = \sqrt{49}$ then $x = 7$

There are 3 methods to solve a quadratic equation that is in the form $ax^2 + bx + c = 0$:

Method 1: Factoring:
1) Get all terms to 1 side of the equal sign. Move them so the squared term is positive
2) Factor your equation
3) Set each factor equal to zero and solve
4) Watch for extraneous solutions (if there is a denominator)

EX: Solve:

a) $6x^2 + x = 12$

b) $x\left(3x+10\right)=77$

c) $\frac{5x}{x-2}+\frac{3}{x}+2=\frac{-6}{x^2-2x}$

Method 2: Using the quadratic formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ ****memorize this formula****

1) must be in the form $ax^2+bx+c=0$
2) –b does not mean it is a negative number – it means the opposite of b
3) when solving the part under the square root (called the determinant), do NOT solve it in parts –use your calculator and key in the entire sequence before pressing the =
4) the determinant can be used to find the number and type of solutions you will have: if it = 0, there will be 1 real solution if it is a positive perfect square, there will be two real rational solutions if it is positve but not a perfect square, there will be two real, irrational solutions if it is negative, there will be no real solution

EX: Solve:

a) $5x^2+13x=6$

b) $x^2-6x-3=0$

c) $\frac{x+1}{3x+2}=\frac{x-2}{2x-3}$

d) **Solve for t:** $s=\frac{1}{2}gt^2+v_0t$

Method 3: Solve by completing the square:

1) Separate the terms: $x^2+bx=c$
2) You will need to add something to both sides that will make the variable side a perfect square binomial (ex: $x^2+2x+1 \Rightarrow (x+1)^2$) To find this "something", you take half of the "b" and square it, then add this number to both sides of the equation.
3) Factor the variable side (it will be (something)^2)
4) Take the square root of both sides
5) Add to get the variable alone.

EX: solve:

a) $x^2+6x+7=0$

b) $x^2-3x=5$

If the x^2 term has a coefficient, divide the entire equation by that term first.

EX: solve: $4x^2-24x=10$

Story problems:

1) A manufacturer of tin cans wishes to construct a right circular cylindrical can of height 20 centimeters and capacity 3000 cm^3. Find the radius inner radius *r* of the can.

2) A box with an open top is to be constructed by cutting 3-inch squares form the corners of a rectangular sheet of tin whose length is twice its width. What size sheet will produce a box having a volume of 60 cubic inches?

3) A baseball is thrown straight upward with an initial speed of 64 ft/sec. The number of feet *s* above the ground after *t* seconds is given by the equation $s = -16t^2 + 64t$.

a) when will the baseball be 48 feet above the ground?

b) when will it hit the ground?

4) The distance that a car travels between the time the driver makes the decision to hit the brakes and the time the car actually stops is called the braking distance. For a certain car traveling *v* mi/hr, the braking distance *d* (in feet) is given by $d = v + \frac{v^2}{20}$.

a) find the braking distance when *v* is 55 mi/hr.

b) if a driver decides to brake 120 feet from a stop sign, how fast can the car be going and still stop by the time it reaches the sign?

5) A rectangular plot of ground having dimensions 26 feet by 30 feet is surrounded by a walk of uniform width. If the area of the walk is 240 square feet, what is its width?

6) A 24-by-36 inch sheet of paper is to be used for a poster, with the shorter side at the bottom. The margins at the sides and top are to have the same width, and the bottom margin is to be twice as wide as the other margins. Find the width of the margins if the printed area is to be 661.5 square inches.

7) A square vegetable garden is to be tilled and then enclosed with a fence. If the fence costs $1 per foot and the cost of preparing the soil is $.50 per square foot, determine the size of the garden that can be enclosed for $120.

8) Two surveyors with two-way radios leave the same point at 9:00 AM, one walking due south at 4 mi/hr and the other due west at 3 mi/hr. How long can they communicate with one another if each radio has a maximum range of 2 miles?

9) A pizza box with a square base is to be made from a rectangular sheet of cardboard by cutting six 1-inch squares from the corners and the middle sections and folding up the sides. If the area of the base if to be 144 square inches, what size piece of cardboard should be used?

10) The speed of the current in a stream is 5 mi/hr. It takes a canoeist 30 minutes longer to paddle 1.2 miles upstream than to paddle the same distance downstream. What is the canoeist's rate in still water?

2.4 Complex numbers:

This is the set of numbers that you use when you have a square root with a negative number under it.

$\sqrt{-1} = i$ so: $i^2 = -1$, $i^3 = -i$, $i^4 = 1$ for example: $\sqrt{-4} = 2i$, $\sqrt{-36} = 6i$ etc

If you are given a complex number with a power, you must simplify it. This is done by dividing out all of the perfect multiples of 4 and evaluating the leftover power:

i^{92} i^{73} i^{43} i^{66}

****Be sure to write your answer in the form $a+bi$ (if there is no real number part, your answer will be $0+bi$**

When working with complex numbers, "pretend" the *i* is a variable until the end, then simplify the complex part:

EX: Add: $(3+4i)+(5+6i)$

Subtract: $(3+4i)-(5+6i)$

Multiply: $(3+4i)(5+6i)$

Divide: $\dfrac{3}{2+4i}$

If you have a negative root, take out the "*i*" first: $\left(5-\sqrt{-9}\right)\left(-1+\sqrt{-4}\right)$

EX: Write in the form $a+bi$:

a) $(7-6i)-(-11-3i)\Rightarrow$

b) $i(2-7i)^2 \Rightarrow$

c) $\dfrac{-4+6i}{2+7i}$

d) $\left(-3+\sqrt{-25}\right)\left(8-\sqrt{-36}\right)\Rightarrow$

Find the values of x and y : Set the real part = real part and complex =complex

$(x-y)+3i=7+yi$

Solve: $x^2-2x+26=0$

Solve: $x^4-3x^2+1=0$

2.5 Other types of equations:

Absolute value:

1) Isolate the absolute value (get it alone on one side of the =)
2) Set up 2 equations: **the stuff inside the abs. value does NOT change
   a) 1st equation: $|\text{stuff}|=\text{number}$
   b) 2nd equation: $|\text{stuff}|=-(\text{number})$
3) Solve both equations
4) **check for extraneous solutions**

EX: solve: $|2x-5|+2=11$

EX: solve: $3|x+1|-2=-11$

Radicals:

1) Isolate the radical
2) Raise both sides to the needed power (if it's a square root, square both sides. If it's a cube root, cube both sides...)
3) Solve for "x"
4) Check for extraneous solutions

EX: Solve: $\sqrt{3-2x}=9$

EX: Solve: $\sqrt{3-x}-x=3$

Rational exponents: 1) Raise both sides to the needed power (reciprocal of the given power)
2) If you have an even root, either in the original power or its reciprocal, watch for "no solution"...If the number you're taking the power of is negative.** see ex. "c"
3) If you are taking the even root of a number, you must have ± in your solution.

EX: Solve:

a) $x^{\frac{5}{3}} = 32$ b) $x^{\frac{4}{3}} = 16$ c) $x^{\frac{2}{3}} = -36$

d) $x^{\frac{3}{4}} = 125$ e) $x^{\frac{3}{2}} = -27$

Quadratics with leading power other than 2:

1) Use the quadratic formula BUT set it equal to the middle variable term and its power
2) Solve the regular old way
3) Take both sides to the needed power

EX: Solve:

a) $x^4 - 3x^2 + 1 = 0$ b) $2x^4 - 10x^2 + 8 = 0$

Prob. Solving 1) Nuclear experiments performed in the ocean vaporize large quantities of salt water. Salt boils and turns into vapor at 1738 K. After being vaporized by a 10-megaton force, the salt takes at least 8 – 10 seconds to cool enough to crystallize. The amount of salt A that has crystallized *t* seconds after an experiment is sometimes calculated using $A = k\sqrt{\frac{t}{T}}$ where *k* and *T* are constants. Solve this equation for *t.*

2) A conical paper cup is to have a height of 3 in. Find the radius of the cone that will result in a surface area of $6\pi\ in^2$.

2.6 Inequalities:

To solve: 1) Use the same method as if you were solving an equation

2) If you multiply or divide by a negative number – you must FLIP THE INEQUALITY SIGN
3) Graph the inequality and find the answer in interval form :
(smallest number possible, biggest number possible)

use parentheses if it is NOT equal to, use brackets if it can be equal to

EX: Solve:

a) $-5x+7\geq 15$

b) $3x-2>14$

c) $4\geq 3x+5>-1$

d) $\dfrac{3}{2x+5}\leq 0$

Inequalities involving absolute value*:*

1) Isolate the absolute value $|\text{stuff}| <> \text{number}$

2) Set up 2 inequalities
   a) 1st ineq: stuff <>number
   b) 2nd ineq: stuff >< opp(number)
3) Solve
4) Graph and find intervals...watch for the AND (overlapping interval) and the OR (everything graphed)

EX: Solve:

a) $|x-4|\leq .03$

$b)\ |x+3|>10$

c) $|5x+2|\leq 0$

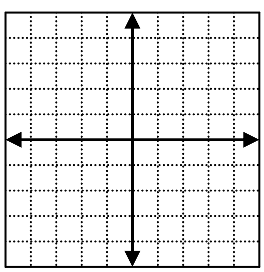
d) $\dfrac{10}{|2x+3|}\geq 5$

Express the statement in terms of an inequality involving an absolute value: The difference of two temperatures T_1 and T_2 within a chemical mixture must be between 5°C and $10^\circ C$.

EX: According to Hooke's law, the force *F* (in pounds) required to stretch a certain spring *x* inches beyond its natural length is given by $F = (4.5)x$. If $10 \le F \le 18$, what are the corresponding values for *x*?

EX: A construction firm is trying to decide which of two models of a crane to purchase. Model A costs $100,000 and requires $8000 per year to maintain. Model B has an initial cost of $80,000 and a maintenance cost of $11,000 per year. For now many years must model A be used before it becomes more economical than B?

3.1 Rectangular Coordinate System



Label: x-axis, y-axis, origin, quadrants
Graph: (3,2) (-3,4) (2,0) (0,4)

DISTANCE FORMULA: Given 2 points (x_1, y_1) and (x_2, y_2), the distance between those points is found using the formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ **memorize this !!!!!**

MIDPOINT FORMULA: Given the same two points, the midpoint between those points is found using the formulas: $x_m = \dfrac{x_1 + x_2}{2}$ and $y_m = \dfrac{y_1 + y_2}{2}$ **memorize this too !!!!!!**

EX: Given A(-1, -3) and B(6, 1), find the distance and the midpoint between them.

APPLICATIONS: You will need to use these formulas to find various other things. They will not tell you to use these formulas; you have to figure that out on your own!

EX: a) Given the vertices of a triangle, find out if it's a right triangle and then, if it is, find its area.

Vertices: A(-6, 3) B(3, -5) and C(-1, 5)

b) Given two points on a line, show that another given point is on its perpendicular bisector:

Points of the line: A(-4, -3) and B(6, 1). Show that the point C(5, -11) is on the perpendicular bisector of $\overline{AB}$

c) Given 1 endpoint and the midpoint of a segment, find the other endpoint: Endpoint A(5, -3) and midpoint C(6, 5), find the other endpoint B.

d) Find all points with coordinates of the form (a,a) that are a distance 3 from the point P(-2, 1)

e) Find a formula that expresses the fact that P(x, y) is a distance 7 from the origin. Describe the set of all such points.

3.2 Graphs of Equations:

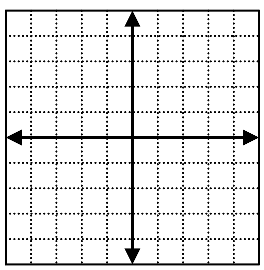
To graph a given equation, you should first recognize what type of graph you'll be working with. We'll work with 3 types this semester:

1) lines: these equations will not have any exponents (ex: 2x + 3y = 5)
2) Parabolas: these are quadratics and will have one square term
   (ex: $x^2 + 2y = 7$ or $2x + 4y^2 = 3$}
3) Circles: these equations will have two terms that are squared, both an "x" and a "y"
   (ex: $(x-3)^2 + y^2 = 25$)

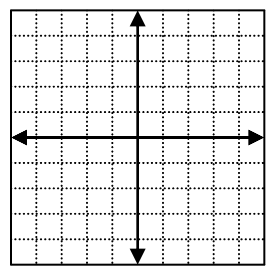
To graph these, at this point, we are going to make T-charts and find the x and y intercepts, if possible:

EX: Graph:

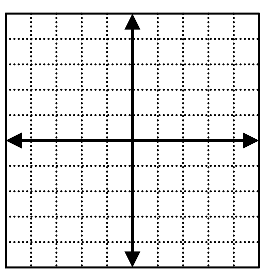
a) $y = -x + 1$



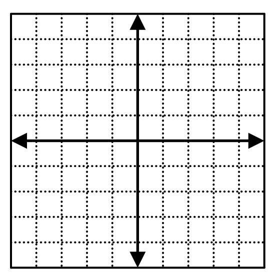
b) $y = x^2 + 2$



c) $y = x^2 - 5$



d) $x = y^2 + 3$



CIRCLES: Circles are different. They have a general equation of the form:

$(x-h)^2+(y-k)^2=r^2$ you guessed it – **memorize this !!**

with the center of the circle at the point (h, k) and having a radius of r

EX:

Find the center and radius of a circle with the equation: $(x-4)^2+(y+2)^2=4$. Graph it.

EX: Write the equation of the circle that satisfies the stated condition:

a) Center at (2, -3) with a radius of 5

b) tangent to both axes, center is in quadrant IV and the radius is 3

c) Center is (0,0) and the circle passes through the point P(4, -7)

d) The endpoints of a diameter are A(4, -3) and B(-2, 7)

Find the center and the radius of the circle with the given equation:

e) $x^2 + y^2 + 4y - 117 = 0$

f) $9x^2 + 9y^2 + 18x - 27y - 9 = 0$

Semi-circles:

Upper or lower half: $y = \pm\sqrt{r^2 - x^2}$ right or left half: $x = \pm\sqrt{r^2 - y^2}$

EX: Find the equations for the left, right, upper, and lower halves of the circle with the equation:

$$(x+3)^2 + y^2 = 64$$

The signal from a radio station has a circular range of 650 miles. A second radio station , located 100 miles east and 80 miles north of the first station has a range of 80 miles. Are there locations where signals can be received from both radio stations?

3.3 Lines

- Standard or general form: Ax + By = C (A, B, and C must be integers)
Slope-intercept form: y = mx + b (m = slope, b = y-intercept)
- Horizontal lines: y = number (slope = 0) ** there will not be an “x” term
- Vertical lines: x = number (slope is undefined) **there will not be a “y” term

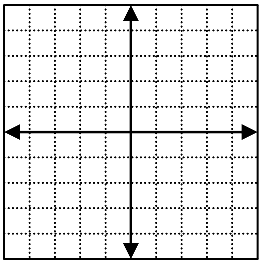
SLOPE: $\dfrac{\text{rYse}}{\text{run}} = \dfrac{y_1 - y_2}{x_1 - x_2}$

- Parallel lines have the same slope
- Perpendicular lines have slopes that are opposite reciprocals

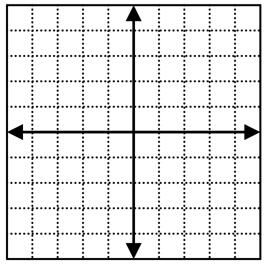
EX: Find the slope and graph the line containing A(-3, 2) and B(5, -4)

EX: Sketch the graph of the line through the point P (-2, 4) with each of the given values of m:

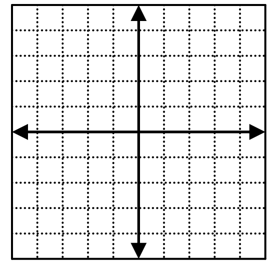
a) m = 1



b) m = -2



c) m = $-\frac{1}{2}$

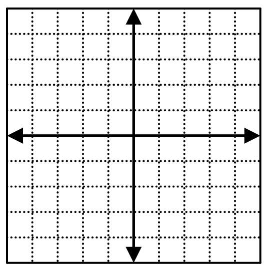


Sketch the graphs of the lines with the following equations:

a) y = -2x – 1

b) y = -2x + 3

c) y = $\frac{1}{2}$ x + 3



To write the equation of a line given certain information, you will need to use the point-slope form of the equation, $\left(y-y_1\right)=m\left(x-x_1\right)$

1) Write the formula
2) Substitute in the values you know (the slope, the given x and y for the x_1 and y_1)
3) solve for y
4) you will get an equation with an x and y in it, manipulate this into the desired form

EX: Find the general form of an equation of the line through A that satisfies the given condition:

a) A(4, 0) with a slope of -3

b) A (-3, 5) parallel to the line x + 3y = 1

c) A (7, -3) perpendicular to the line $2x - 5y = 8$

d) through A (5, 2) and B (-1, 4)

e) with an x-intercept of -5 and a y-intercept of -1

f) through A (5, -2) parallel to the y-axis

g) through A (5, -2) perpendicular to the y-axis

h) Given A (3, -1) and B (-2, 6), find the equation for the perpendicular bisector of $\overline{AB}$

i) Given $x - 5y = -15$, find the slope, y-intercept, and graph the line

j) Find an equation for the line that bisects quadrants I and III

1) A baby weighs 10 pounds t birth, and three year later the child's weight is 30 pounds. Assume that childhood weight W in pounds is linearly related to age *t* in years.

a) express W in terms of t

b) What is W on the child's sixth birthday?

c) At what age will the child weigh 70 pounds?

2) A college student receives an interest-free loan of $8250 from a relative. The student will reapy $125 per month until the loan is paid off

a) express the amount P (in dollars) remaining to be paid in terms of time t (in months)

b) After how many months will the student owe$5000?

3) In 1870 the average ground temperature in Paris was 11.8 degrees Celsius. Since then it has risen at a nearly constant rate, reaching 13.5 degrees in 1969.

a) express the temperature T in terms of time t where t = 0 corresponds to the year 1870 and $0 \le t \le 99$

b) During what year was the average ground temperature 12.5 degrees?

4) The relationship between the temperature reading *F* on the Fahrenheit scale and the temperature reading *C* on the Celsius scale is given by $C = \frac{5}{9}(F - 32)$. Find the temperature at which the reading is the same on both scales AND when is the Fahrenheit reading twice the Celsius reading?

3.4 Functions

To be a function:

- No x is repeated in the domain (the set of all x values that work for the function)
- Y's may be repeated in the range (the set of all y values that work for the function)
- The graph passes a vertical line test (a vertical line will not touch the graph in 2 points at the same time
- It's written f(x) = ... f(x) is just cool math nerd language for "y"

The **domain:** The set of all x values that make the function work – it's usually the set of all real numbers with these exceptions:

- Radicals: The "stuff" under the radical must be non-negative: set the "stuff" ≥ 0 and solve
- Variables in the denominator: the "stuff" in the denominator cannot equal zero: set the "stuff" $\ne 0$ and solve.

EX: Find the domain:

a) $f(x) = \sqrt{8 - 3x}$

b) $f(x) = \dfrac{\sqrt{2x - 3}}{x^2 - 5x + 4}$

You may be asked to look at graphs and find certain information:

Ex: Look at the graph and find:(with the exceptions of the range, all questions are about the x values)

a) the domain:______________________

b) the range:________________________

c) increasing?_______________________

d) decreasing:_______________________

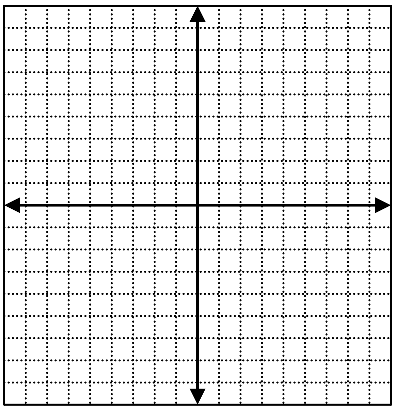
e) constant:_______________________

f) find f(1):________________________

find all x such that:

f) f(x) = 1_________________________

g) f(x)>1___________________________



Finding function values: Substitute in the value in the given parenthesis for every variable in the original function

EX: Given f(x) = 5x – 2, find:

a) f(2) b) f(-3) c) f(a)

EX: Given $f(x) = -x^3 - x^2 + 3$, find:

a) f(3) b) f(0) c) f(-2)

EX: Given f(x)= 4 – 3x, find:

a) f(a) b) f(-a) c) –f(a)

d) f(a + h) e) f(a) + f(h)

f) $\dfrac{f(a+h)-f(a)}{h}$

EX: Given g(x) = 2x – 5, find:

a) $g\left(\dfrac{1}{a}\right)$

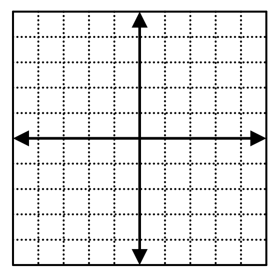
b) $\dfrac{1}{g(a)}$

c) $g(\sqrt{a})$

d) $\sqrt{g(a)}$

EX: Given $f(x) = 4 - x^2$, find:

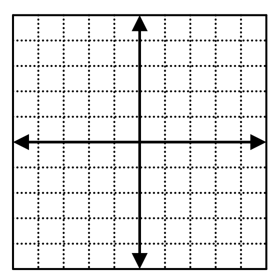
Graph:



a) the domain:____________________________
b) the range:______________________________
c) increasing?______________________________
d) decreasing?____________________________
e) constant?_______________________________

EX: Given: $f(x) = \sqrt{16 - x^2}$, find:

Graph:



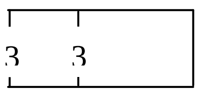
a) the domain:____________________________
b) the range:______________________________
c) increasing?______________________________
d) decreasing?____________________________
e) constant?_______________________________

EX: If a linear function, *f*, satisfies the given conditions, find *f(x)*. *f(-2) = 7 and f(4) = -2*

Applications:

1) From a rectangular piece of cardboard having dimensions 20 in by 30 in, an open box is to be made by cutting out an identical square area of x^2 from each corner and turning upt he sides. Express the volume V of the box as a function of x.

2) A small office unit is to contain 500 square feet of floor space. Express the length *y* of the building as a function of the width *x*.



b) If the walls cost $100 per running foot, express the cost, *C,* as a function of the width *x.*

3) For children between the ages of 6 and 10, height *y* (in inches) is frequently a linear function of age *t* (in years). The height of a certain child is 48 inches at age 6 and 50.5 inches at age 7.

A) Express *y* as a function of *t*.

b) Predict the height of the child at age 10.

4) An aquarium of height 1.5 ft is to have a volume of 6 cubic ft. Let *x* denote the length of the base and *y* the width.

a) express *y* as a function of *x*

b) express the total number *S* of square ft of glass needed as a function of *x.*

5) For children between the ages of 6 and 10, height *y* (in inches) is frequently a linear function of age *t* (in years). The height of a certain child is 48 inches at age 6 and 50.5 inches at age 7. a) express *y* as a function of *t*.

b) sketch the line and interpret the slope.

C) predict the height of the child at age 10.

6) A hot air balloon is released at 1 PM and rises vertically at a rate of 2m/sec. An observation point is situated 100 meters from a point on the ground directly below the balloon. If *t* denotes the time in seconds after 1 PM, express the distance *d* between the balloon and the observation point as a function of *t.*

3.5 Graphs of functions:

To find out if a function is **even** or **odd**: Substitute in (-x) for x in the original function, simplify this expression and compare the two answers. The function is:

- Even if f(x) = f(-x)
- Odd if f(x) and f(-x) are total opposites
- Neither if f(x) and f(-x) has some terms that are the same and some that are the opposite.

EX: Identify the function as even or odd:

a) $f(x) = 5x^3 + 2x$ b) $f(x) = \sqrt{x^2 + 4}$ c) $f(x) = 8x^3 - 3x^2$

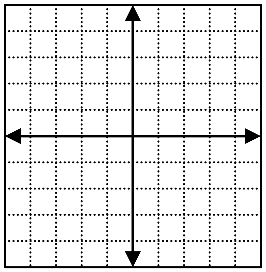
Graph shifting: You will be asked to take an original function and shift it left, right, up, or down, and compress it or expand it. To do this, you:

1) write an original T-chart for the original function
2)Make a new T-chart with the changes to x and y according to the new function

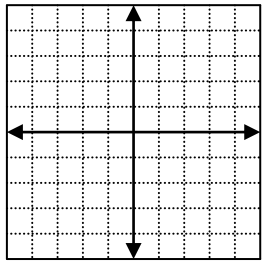
- If the change is huddled inside with the "x" the shift affects the x value using the hopposite hoperation
    - Ex: original function: $f(x) = x^2$
    - New function: $f(x) = (x+1)^2$ will change all the x values by subtracting 1 from the original x values
- If the change is tagging along and is not in with the "x", the shift affects the y value using exactly the operation as it is written
    - Ex: original function: $f(x) = x^2$
    - New function: $f(x) = x^2$ + 1 will change the y values by adding 1 to each of the original y values

EX: Given the function $f(x) = |x|$, graph the new functions:

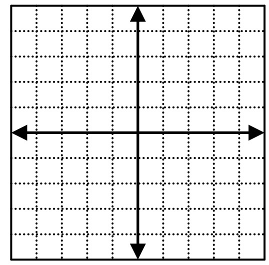
$f(x)$



a) $f(x) = |x| - 3$

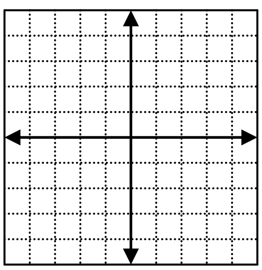


b) $f(x) = |x - 3|$

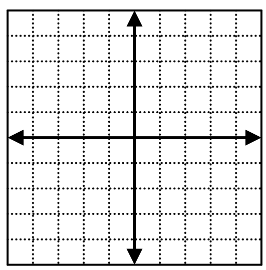


EX: Given $f(x) = 2x^2$, graph the new functions:

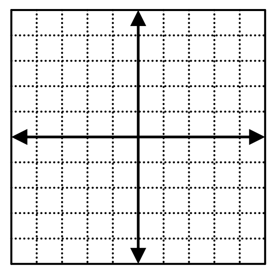
$f(x)$



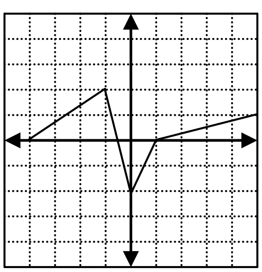
a) $f(x) = 2x^2 - 4$



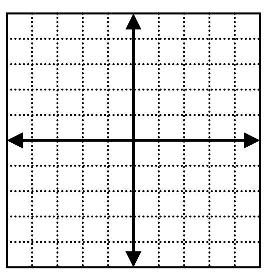
b) $f(x) = 2(x+1)^2$



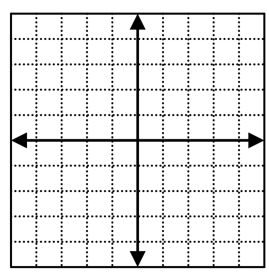
EX: Given the graph of f(x), graph:



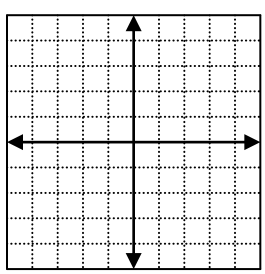
a) f(x + 2)



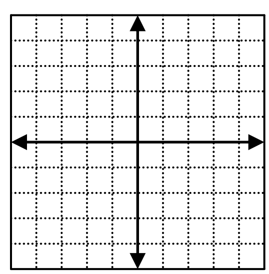
b) f(x) – 2



c) -2f(x)



d) f(-2x)



Determine new coordinates for f(x) given the new function:

P(3, -6)

a) y = f(x) + 7

b) y = 2f(x + 1)

c) d) y = -4f(2x) – 3

d) $y = \frac{1}{2} f\left(\frac{1}{3}x\right) + 2$

What's happening to the graph? Look at each part of the equation and decide what has happened to both the original x and y

EX: y = f(x + 2) – 3

b) y = -f(x)

c) y = f(-2x) + 1

d) y = - $\frac{1}{3}$ f(x – 3)

Finding the domain and range of a "new function"

Given: Domain:[-8, -3] and R: [-2, -4] for f(x), find the domain and range for:

a) $y = \frac{1}{2} f(x)$

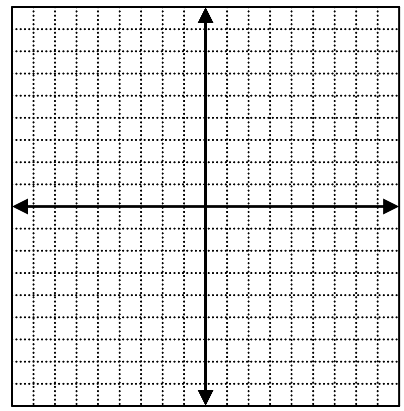
b)$y = 3f(x)$

c) $y = (x-3)+2$

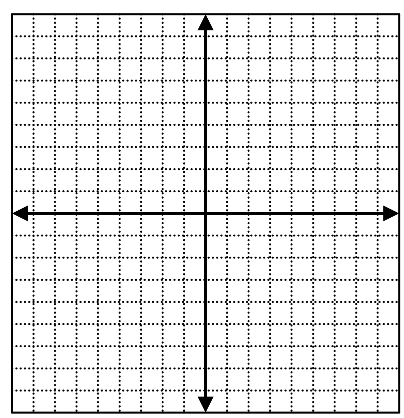
d) $y = f(-x)$

Piecewise graphs: 1) Split the coordinate grid into parts according to the given boundaries:
2) Make a T-chart for each section of the graph. Use the beginning and ending values for each section
3) Only graph the point in between the boundary lines

EX: Graph: $f(x)=\begin{cases} 3 & \text{if } x<-2 \\ -x+1 & \text{if } -2\le x<2 \\ -3 & \text{if } x>2 \end{cases}$



EX: $f(x)=\begin{cases} x-3 & \text{if } x\le -3 \\ -x^2 & \text{if } -3<x<1 \\ -x+4 & \text{if } x\ge 1 \end{cases}$



EX: Write a piece-wise function that show the following:

1) A telephone company charges 25 cents for a long-distance call that does not exceed one minute; for longer calls it charges 15 cents for each additional minute. Find a piece-wise function that specifies the total cost of a long-distance call of x minutes

2) A certain country taxes the first $17,000 of an individual's income at a rate of 105, and all income over $17,000 is taxed at 25%. Find a piecewise-defined function T that specifies the total tax on an income of x dollars. Simplify your answer completely.

3) A certain state taxes the first $400,000 in property value at a rate of 1%; all value over $400,000 is taxed at 1.75%. Find a piecewise-defined function T that specifies the total tax on a property valued at x dollars.

4) A certain paperback sells for $12. The author is paid royalties of 10% on the first 10,000 copies sold, 12.5% on the next 5000 copies sold, and 165on any additional copies. Find a piecewise-defined function R that specifies the total royalties if x copies are sold.

5) An electric company charges its customer's \$0.0575 per kilowatt-hour for the first 1000 kWh used, \$0.0533 for the next 4000 kWh, and \$0.0513 for any kWh over 5000. Find a piecewise-defined function C for a customer's bill of x kWh.

3.6 Quadratic functions:

- Graph will be a **parabola**
- One form of a quadratic is ax^2+bx+c
- The standard form of a parabola with a vertical axis is $y=a\left(x-h\right)^2+k$, where (h,k) is the vertex of the parabola and a tells you if it opens up (if "a" is positive) or down (if a is negative)
- X value of the vertex can be found using: $x=\dfrac{-b}{2a}$
- To find the y value of the vertex, substitute this back into the original equation and solve for y

- The y value of the vertex is called the maximum (if the parabola opens down) or the minimum (if the parabola opens up) of the parabola

- To find the **x – intercepts** (where the graph crosses the x-axis) you will use the quadratic formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$. Then you can use the x-intercepts and the vertex to graph the parabola. Some parabolas will not cross the x-axis so you will have to use a T-chart to graph.

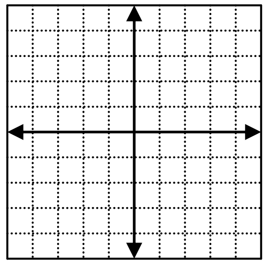
EX: Express *f(x)* in standard form:

a) $f(x)=2x^2-12x+22$

b) $f(x)=-3x^2+24x-50$

EX: Consider the function $f(x) = -3x^2 - 6x - 6$. Sketch a graph.

a) Find the zeros of *f.*



b) Find the x- and y- intercepts of *f*.

c) Find the maximum or minimum value of *f*.

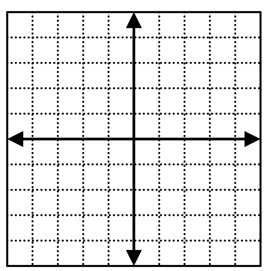
d) Find the domain and range of *f*.

e) find the intervals on which *f* is increasing or decreasing.

f) find the equation of the AOS

EX: Consider the function. $f(x) = x^2 - 6x + 5$

a) Find the zeros of *f.*



b) Find the x- and y- intercepts of *f*.

c) Find the maximum or minimum value of *f*.

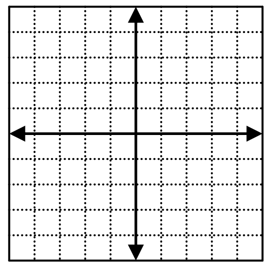
d) Find the domain and range of *f*.

e) find the intervals on which *f* is increasing or decreasing.

f) find the equation of the AOS

Consider the function. $f(x) = 9x^2 + 24x + 16$

a) Find the zeros of *f.*



b) Find the x- and y- intercepts of *f*.

c) Find the maximum or minimum value of *f*.

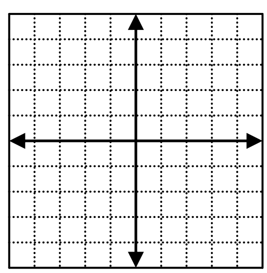
d) Find the domain and range of *f*.

e) find the intervals on which *f* is increasing or decreasing.

f) find the equation of the AOS

Consider the function. $f(x) = -3x^2 - 6x - 6$

a) Find the zeros of *f.*



b) Find the x- and y- intercepts of *f*.

c) Find the maximum or minimum value of *f*.

d) Find the domain and range of *f*.

e) find the intervals on which *f* is increasing or decreasing.

f) find the equation of the AOS

Find the standard form of the parabola that has a vertical axis and satisfies the given conditions:

a) vertex: (0, 5) passing through (2, -3)

b) x-intercepts 8 and 0, lowest point has y-coordinate -48

An object is projected vertically upward with an initial velocity of v_o ft/sec, and its distance $s(t)$ in feet above the ground after *t* second sis given by the formula $s(t) = -16t^2 + v_o t$. If the object hits the ground after 12 seconds, find the initial velocity v_o. Find its maximum distance above the ground.

Flights of leaping animals typically have parabolic paths. The length of a frogs leap is 9 feet and the maximum height off the ground is 3 feet. Find a standard equation for the path of the frog.

A farmer wishes to put a fence around a rectangular field and then divide the field into 2 rectangular plats by placing 2 fences parallel to one of the sides. IF the farmer can only afford 1000 yards of fencing, what dimensions will give the maximum rectangular area?

3.7 Operations of Functions:

(f + g) (x) = f(x) + g(x)

(f – g) (x) = f(x) – g(x)

(fg) (x) = f(x) · g(x)

$$\frac{\mathrm{f}}{\mathrm{g}}(x)=\frac{\mathrm{f(x)}}{\mathrm{g(x)}}$$

EX: Given f(x) = x + 3 and g(x) = x^2 , find:

(f + g)(x) (f – g)(x) (fg)(x) (f/g)(x)

composite functions:

They will be written in weird math nerd form:

$$\begin{aligned}&(f\circ g)(x)\text{ changes to }f[g(x)]\\&(g\circ f)(x)\text{ changes to }g[f(x)]\\&(f\circ f)(x)\text{ changes to }f[f(x)]\\&(g\circ g)(x)\text{ changes to }g[g(x)]\end{aligned}$$

You must work from the inside out (cover up the outside, do the inside 1st, add it the outside)
EX: Given: $f(x)=2x-1$ and $g(x)=-x^2$, find:

a) f[g(x)] b) g[f(x)]

c) f[f(x)] d) g[g(x)]

EX: Given $f(x) = 3x - 1$ and $g(x) = 4x^2$, find:

a) $(f \circ g)(x)$

b) $(g \circ f)(x)$

c) $f[g(-2)]$

d) $g[f(-3)]$

EX: Given $S(r) = 4\pi r^2$ and $D(t) = 2t + 5$, find $(SoD)(t)$

Solve: Given $f(x) = x^2 - x - 2$ and $g(x) = 2x - 1$, solve $(f \circ g)(x) = 0$

Solve: A hot-air balloon rises vertically from ground level as a rope attached to the base of the balloon is released at the rate of 5 ft/sec. The pulley that releases the rope is 20 ft from a platform where passengers board the balloon. Express the altitude *h* of the balloon as a function of time *t.*

Composite functions using charts: Find the requested composition using the given chart:

X	0	1	2	3
T(x)	2	3	1	0

X	0	1	2	3
S(x)	1	0	3	2

Find: a) T[S(1)]   b) S[T(1)]   c) t[T(1)]   d) S[S(1)]

2.7 Solving a quadratic inequality:

1) Get the inequality compared to "0"
2) Factor the quadratic
3) Find the zeros (set each factor = 0 and solve)
4) Set up a resulting sign chart
5) Find the intervals that "work"

EX: Solve:

a) $x^2 + 4x \geq -3$

b) $(3x+1)(5-10x) > 0$

c) $(x-5)(x+3)(-2-x)<0$

d) $x^2>9$

e) $\dfrac{x-2}{3x+5}\le 4$

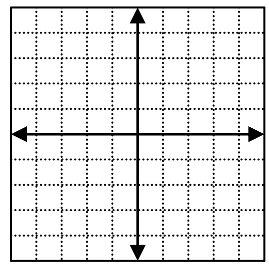
f) $-\dfrac{1}{5}(x+2)^2(x+1)\ge 0$

4.1 Graphs of polynomial functions of degree greater than 2

1) Set up a resulting sign chart
2) find the intervals that are above (> 0)and below (< 0) the x-axis
3) SKETCH a graph of the curve using this information.

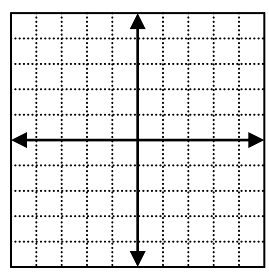
EX: Find all values of x such that f(x) > 0 and all x such that f(x) < 0. Sketch the graph

a) $f(x) = -x^3 + 3x^2 + 10x$



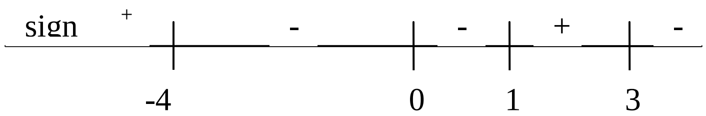
- find the zeros of *f*
- find the y-intercepts of *f*
- find the domain and range of *f*
- on what interval(s) is *f(x)>0?*
- on what interval(s) is *f(x)<0?*
- Is the function even, odd, or neither?

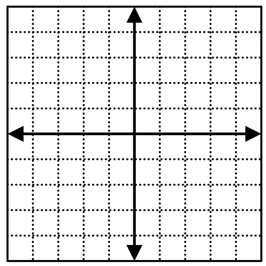
b) $f(x) = -\frac{1}{8}(x+4)(x-2)(x-6)$



- find the zeros of *f*
- find the y-intercepts of *f*
- find the domain and range of *f*
- on what interval(s) is *f(x)>0?*
- on what interval(s) is *f(x)<0?*
- Is the function even, odd, or neither?

c) Use the sign chart to graph:





EX: If $f(x) = kx^3 + x^2 - kx + 2$, find k such that *f* contains the point (2, 12).

***Substitute in 2 for x and 12 for y, solve for k

Prob solving: From a rectangular piece of cardboard having dimensions 20 in x 30 in, an open box is to be made by cutting out identical squares of area x^2 from each corner and turning up the sides.

a) find the volume of the box.

B) find all positive values of *x* such that $V(x) > 0$.

c) Sketch the graph.

4.6 Variation:

You will see the word "varies" or "is proportional to". This tells you to set up a general formula using the variables involved and a constant of proportionality, k.

- If it varies **directly** or **is directly proportional to**, put your information on top
- If it varies **inversely**, put your information on the bottom
- There is only one top and one bottom per problem.

Express the statement as a formula that involves the given variables and a constant of proportionality *k*, and then determine the value of *k* from the given conditions.

1) r varies directly as s and inversely as t. If s = -2 and t = 4, then r = 7

2) r is directly proportional to the product of s and v and inversely proportional to the cube of p. If s = 2, v = 3, and p = 5, then r = 40.

3) y is directly proportional to the square of x and inversely proportional to the square root of z. If x = 5 and z = 16, then y = 10

4) Hooke's law states that the force *F* require to stretch a spring *x* units beyond its natural length is directly proportional to *x*.

a) express *F* as a function of *x* by means of a formula that involves a constant of proportionality *k.*

b) A weight of 4 pounds stretches a certain spring from its natural length of 10 inches to a length of 10.3 inches. Find the value of *k* in part a.

c) What weight will stretch the spring in part b to a length of 11.5 inches?

d) sketch a graph of the relationship between *F* and *x* for $x \geq 0$

5) The ideal gas law states that the volume *V* that a gas occupies is directly proportional to the product of the number *n* of moles of gas and the temperature *T* (in K) and is inversely proportional to the pressure *P* (in atmospheres).

a) express *V* in terms of *n, T, P* and a constant of proportionality *k*

b) What is the effect on the volume if the number of moles is doubled and both the temperature and the pressure are reduced by a factor of one-half?

9.1 Systems of equations:

You will be solving 2 equations at the same time. There will be two variables, usually x and y, and you need to solve for each of them. You are looking for the point or points of intersection – the point(s) that will work for both equations.

There are two methods of solving systems: The first method can be used with any types of equations. It is called **substitution.**

To solve using substitution:

1) Solve one of the equations for one of the variables. Try to solve for the "easiest" variable – the one without a power and without a coefficient, if possible.
2) Substitute that expression into the other equation
3) Solve
4) Substitute this back into the 1^{st} equation and solve.
5) Check each pair for extraneous solutions

EX: Solve:

a) $\begin{cases} y^2 = 1 - x \\ x + 2y = 1 \end{cases}$

b) $\begin{cases} x^2 + y^2 = 16 \\ 2y - x = 4 \end{cases}$

c) $\begin{cases} xy = 2 \\ 3x - y + 5 = 0 \end{cases}$

d) $\begin{cases} y^2 - 4x^2 = 4 \\ 9y^2 + 16x^2 = 140 \end{cases}$

e) The perimeter of a rectangle is 40 inches and its area is 96 square inches. Find its length and width.

f) Sections of cylindrical tubing are to be made from thin rectangular sheet that have an area of 200 square inches. Is it possible to construct a tube that has a volume of 200 cubic inches? If so, find *r* and *h.*

9.2 Solving systems using elimination

Elimination can only be used to solve systems that have the exact same 2 variable parts. (For example, the variables could both be x and y, x^2 and y^2 **.**

The mission in elimination is to get one of the variable terms to have exact opposite coefficients (you may need to multiply to accomplish this), then you:

- add the equations- which eliminates a variable
- Solve this new equation
- Substitute this value into either of the two original equations and solve for the other variable.

EX: Solve:

a) $\begin{cases} 3x + y = 4 \\ 2x - y = 6 \end{cases}$

b) $\begin{cases} 2x + 3y = 2 \\ x - 2y = 8 \end{cases}$

c) $\begin{cases} 7x - 8y = 9 \\ 4x + 3y = -10 \end{cases}$

d) $\begin{cases} \frac{1}{3}c + \frac{1}{2}d = 5 \\ c - \frac{2}{3}d = -1 \end{cases}$

e) $\begin{cases} 3m - 4n = 2 \\ -6m + 8n = -4 \end{cases}$

f) A man rows a boat 500 feet upstream against a constant current in 10 minutes. He then rows 300 feet downstream (with the same current) in 5 minutes. Find the speed of the current and the equivalent rate at which he can row in still water.

g) A silversmith has two alloys, one containing 35% silver and the other 60% silver. How much of each should be melted and combined to obtain 100 grams of an alloy containing 50% silver?

h) If an object is projected vertically upward from an altitude of s_0 feet with an initial velocity of v_0 ft/sec, then its distance *s(t)* above the ground after *t* seconds is $s(t) = -16t^2 + v_0 t + s_0$. If *s*(1)=84 and *s*(2)=116, what are v_0 and s_0?

5.1 INVERSE FUNCTIONS:

Given a function, its inverse function is the set of all points where the domain of the original function becomes the range of the inverse, and the range of the original function becomes the domain of the inverse function. In other words, the "x values" and the "y values" switch.
So, for example, if the original function contains the point (2,3) the inverse function contains the point (3,2)

In order to have an inverse, the original function must be a one-to-one function. It must pass the horizontal line test or it must pass the algebraic test to see if f(a)=f(b). To do this you:

1) substitute in a for x and simplify
2) substitute in b for x and simplify
3) compare the results, you should get a = b

EX: Are the following functions 1-1?

a) $f(x) = 3x - 7$

b) $f(x) = x^2 - 9$

To find the inverse of a function, exchange the x and y, and then solve for the new y.

EX: Find the inverse of:

a) $f(x) = 7 - 2x$

b) $f(x) = \dfrac{1}{3x-2}$

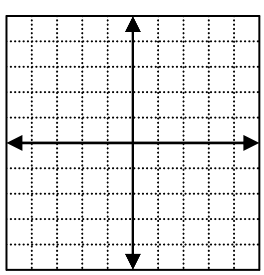
c) $f(x)=5x^2+2, x\geq 0$

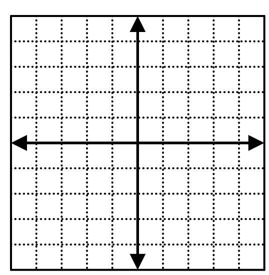
d) $f(x)=\sqrt{4-x^2},\ 0\leq x\leq 2$

e) $f(x)=\dfrac{3x+2}{2x-5}$

Remember: with a function and its inverse the domain and range switch.
And the curve reverses

EX: Given f(x) with D = [-3, 3] and R =[-2,2], find and graph f^{-1}.



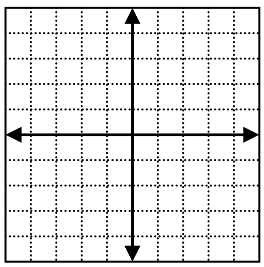


5.2 Exponential functions:

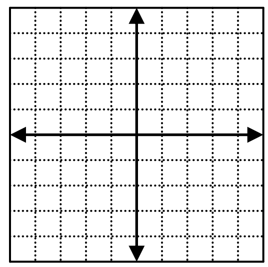
- The variable will be in the exponent (ex: 5^x)
- To graph these, you will make a T-chart usually using the values of -2, 0, 2 for x
- The graph will have a horizontal asymptote – this is a horizontal line that the graph approaches, but never touches.
- The asymptote will be the x-axis **unless** there is a number "tagging along" on the equation, and then the asymptote will be moved up or down to that number.

EX: graph:

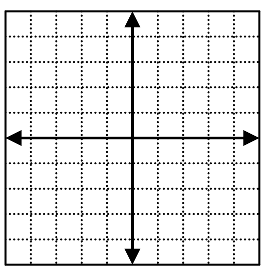
a) $f(x) = 4^x$



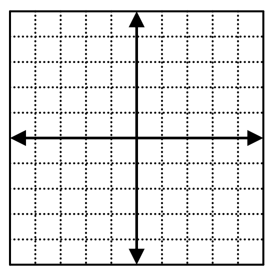
b) $f(x) = 4^{x-2}$



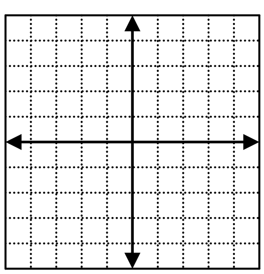
c) $f(x) = 3(4)^x$



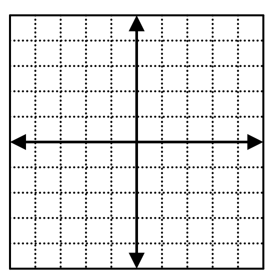
d) $f(x) = 4^x + 3$



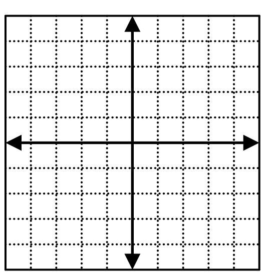
e) $f(x) = 4^x - 3$



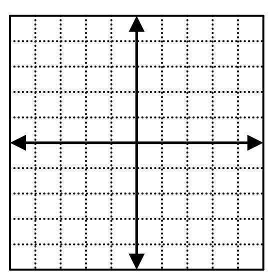
f) $f(x) = 4^{-x}$



g) $f(x) = 4^{3-x}$



h) $f(x) = \left(\frac{1}{4}\right)^x$



To solve exponential equations:

- You must have the same base
- Set the exponents equal to each other
- Solve

EX: Solve:

a) $7^{x+6} = 7^{3x+4}$

b) $9^{x+5} = 27^{2x-1}$

c) Write the equation of the form $y = ba^x$ with a base of 9 going through the point (2, 1)

d) Consider the function $f(x) = -\left(\frac{1}{2}\right)^x + 4$. Sketch a graph of the function:

- Find the domain and range of the function.

- Find the zeros and the y-intercept of the function.

- Find an equation for the horizontal asymptote.

- ON what interval(s) is $f(x) > 0$ and on what interval(s) is $f(x) < 0$?

- Find the interval(s) on which *f* is increasing or decreasing. Is *f* odd, even, or neither?

Compound interest:

- P = principal (original money)
- A = amount you end up with
- r = rate of interest (given as a %, change to a decimal)
- n = number of times per year interest is calculated
- t = time (in years)

compound interest formula: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ **this will be given to you on a test !!**

EX: If $1000 is invested at a rate of 12% per year compounded monthly, find the principal after 6 months.

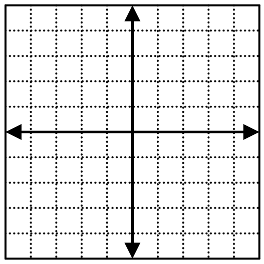
EX: If a savings fund pays interest at a rate of 10% compounded semiannually, how much money invested now will amount to $5000 after 1 year?

5.3 The natural exponential function

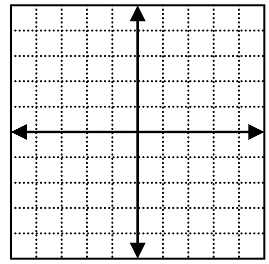
- The base is e, which is a constant approximately equal to 2.71828
- The natural exponential function f is defined by $f(x) = e^x$
- To graph, make a T-chart, and then punt☺

EX: Graph:

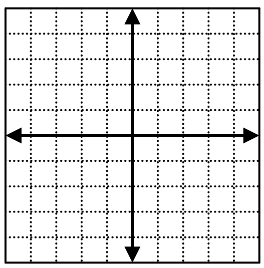
a) $f(x) = e^{x+4}$



b) $f(x) = e^{2x}$



c) $f(x) = -2e^{x}$



To find zeros of an exponential function – factor and then set each factor equal to zero, and solve. ***remember $e \neq 0$

EX: Solve:

a) $f(x) = -x^2e^x + 2xe^x$

b) $f(x) = x^3\left(4x^{3x}\right) + 5x^2e^{3x}$

Continuously compounded interest: $A = Pe^{rt}$ (again, this will be given to you)

EX: a) $20,000 is deposited in a money market account that pays interest at a rate of 8% per year compounded continuously. Determine the balance in the account after 5 years.

b) How much money, invested at an interest rate of 11% per year compounded continuously, will amount to $100,000 after 18 years?

5.4 Logarithmic Functions

- The log function is the inverse of the exponential function
- The log is the exponent in the exponential equation form
- You can switch between the two functions very easily
- You'll need to switch between the forms to solve future equations.
- You cannot take the log of a negative number or have a negative base. The exponent is the only part of the equation that can be negative.

Log form: $\log_{base}(\text{thing}) = \text{exponent}$ Exp form: $\text{thing} = base^{\text{exponent}}$

EX: Change to log from:

a) $3^5 = 243$ b) $3^{-4} = \frac{1}{81}$ c) $c^p = d$

d) $7^x = 100p$ e) $3^{-2x} = \frac{p}{f}$ f) $(.9)^x = \frac{1}{2}$

EX: Change to exponential form:

a) $\log_2 32 = 5$ b) $\log_3 \frac{1}{243} = -5$ c) $\log_t x = p$

d) $\log_3 (x+2) = 5$ e) $\log_2 m = 3x+4$ f) $\log_b 512 = \frac{3}{2}$

There are two special log functions:

- The **common log** is written without a base, but has a base of 10 assumed $\log x = 2$
- The **natural log** has a base of e and is written $\ln x = 2$

To solve or find a missing number, you switch back and forth between the log form and the exponential form. In order for an equation to be in log form, "log" must be the **first** thing in the equation (look at ex. e)

There are 4 shortcuts you might want to learn:

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a ax = x$
- $a^{\log_a x} = x$

EX: Find the number, if possible

a) $\log_8 1$ b) $\log_9 9$ c) $\log_5 0$

d) $\log_6 6^7$ e) $5^{\log_5 4}$ f) $\log_3 243$

g) $10^{\log 3}$

h) $\log 10^{5}$

i) $\log 100$

j) $\log(.0001)$

k) $e^{\ln 2}$

l) $\ln e^{-3}$

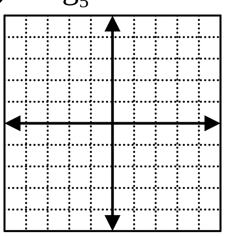
m) $e^{2+\ln 3}$

The graph of the log function has a vertical asymptote, which will be the y-axis unless there is a "huddled" shift, then the asymptote shifts the hopposite way of the operation stated.

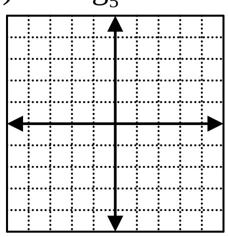
Again, we'll make a T-chart and punt!

EX: Sketch:

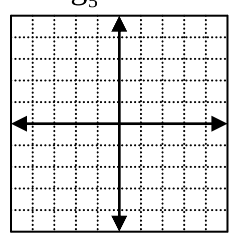
a) $f(x) = \log_5 x$



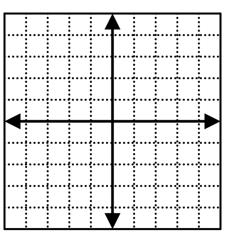
b) $f(x) = -\log_5 x$



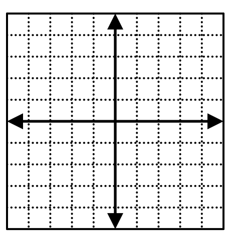
c) $f(x) = 2\log_5 x$



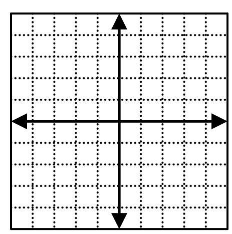
d) $f(x) = \log_5(x+2)$



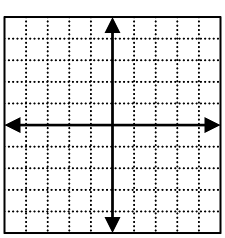
e) $f(x) = (\log_5 x) + 2$



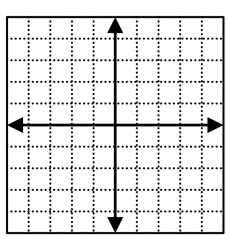
f) $f(x) = \log_5(x-2)$



g) $f(x) = \log_5(-x)$



h) $f(x) = \log_5(3-x)$



EX: Using your calculator: The calculator will only do logs that are common (have a base of 10) or natural (have a base of *e*).

Ig you're given the log and need to find what number it is the log of, you need to switch to exponential form first, then key it in the calculator.

Use your calculator to approximate the logs to three significant digits:

a) log x = 4.9680

b) log x = -2.2118

c) ln x = .95

d) ln x = -5

Solve:

a) $\log_3(x+4)=\log_3(1-x)$

b) $\log_7(x-5)=\log_7(6x)$

c) $\log_3(x-4)=2$

d) $\log_4 x=-\frac{3}{2}$

5.5 Properties of logs:

To **expand**:

- $\log_a(uw)=\log_a u+\log_a w$
- $\log_a\left(\frac{u}{w}\right)=\log_a u-\log_a w$
- $\log_a(u)^c=c\log_a u$

EX: Express in terms of logs of x, y, z, or w:

a) $\log_3(xyz) \Rightarrow$

b) $\log_3\left(\dfrac{xz}{y}\right) \Rightarrow$

c) $\log_3 \sqrt[5]{y} \Rightarrow$

d) $\log_3 \dfrac{x^3 w}{y^2 z^4} \Rightarrow$

e) $\ln \sqrt[3]{\dfrac{y^4}{z^5}} \Rightarrow$

To **condense:** If the log has a "+" in front of it, it goes on top, if it has a "-" in front, it goes on the bottom:

EX: Express as 1 log:

a) $\log_3 x + \log_3(5y) \Rightarrow$

b) $\log_3(2z) - \log_3 x \Rightarrow$

c) $5\log_3 y \Rightarrow$

d) $2\log\frac{y^3}{x} - 3\log y + \frac{1}{2}\log x^4 y^2 \Rightarrow$

To solve equations involving logs:

1) If every term has a log, get each side into the log ______ = log _______, and then "cancel" the logs and solve
2) If there is 1 term WITHOUT a log, get all logs on one side and condense them into one log. Then you'll have log _____= #. Switch to exponential form and solve.

*********3) Check your solution. You CANNOT have log(-#)

EX: Solve:

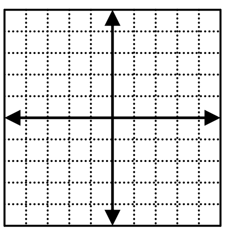
a) $\log_6(2x-3) = \log_6 12 - \log_6 3$

b) $\log x - \log(x+1) = 3\log 4$

c) $\log_6(x+5)+\log_6 x=2$

d) $\ln x = 1+\ln(x+1)$

Graph: $f(x)=\log_2 \sqrt[3]{x}$



5.6 Exponential and Log equations:

- Exponential equations – Take the log of both sides, manipulate using log properties
- Log equations – use the change of base formula: $\log_b u = \dfrac{\log u}{\log b}$. This will get you into common logs and you can then use your calculator:

EX: Solve. Give an exact solution and a 3-decimal approximation:

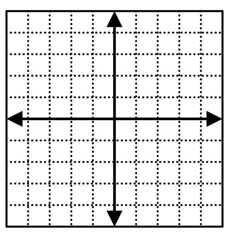
a) $3^x = 21$

b) $3^{2-3x} = 4^{2x+1}$

c) $\log_5 6$

d) $\log(x^2) = (\log x)^2$

EX: Given $f(x) = 3^x - 6$, graph the function and approximate the x and y-intercepts.



THE END THE END THE END THE END THE END THE END THE END

Practice for exam 1:

A: Simplifying exponents:

1) $\left(\frac{3x^{-2}y^4}{x^3y^2}\right)^{-2}$ 2) $\frac{\left(4x^2y^2\right)\left(2y\right)^{-3}}{xy^{-4}}$ 3) $\left(xy^{-1}\right)\left(2x^2y\right)^{-3}$ 4) $\left(\frac{5x^2y^{-3}}{x^0y^{-2}}\right)^{-2}$

B: Solve for the requested variable:

5) $m=\frac{t^2}{t+rt}$, for r (6) $A=\frac{h(b+c)}{2}$, for b (7) $A=L^2+\frac{1}{2}bh$, for h (8) $A=P\left(1+\frac{r}{k}\right)$, for k

C: Factor completely:

9) $16x^6y-81x^2y$ 10) $3x^3+6x^2-15x-30$ 11) $16x^3+12x^2-10x$

12) $2x^5-3x^4-32x+48$ 13) $15x^2-18x-24$ 14) x^2-49x^4

15) $4x^2+38x-42$ 16) $243y^4-48$

D: Simplify:

17) $\dfrac{\frac{1}{ab}-\frac{b}{a}}{\frac{1}{a^2b}+\frac{1}{a^2}}$ 18) $\dfrac{\frac{1}{a}-b}{\frac{1}{b}-a}$ 19) $\dfrac{x-2}{1-\frac{4}{x+2}}$ 20) $\dfrac{a(b-a)}{a^2-ab}$

E: Solve:

22) $\frac{x}{3x+5}-\frac{2}{3x-5}=\frac{3x^2-2x+5}{9x^2-25}$ 23) $\frac{4}{x-1}-\frac{9}{x+1}=\frac{3x+2}{x^2-1}$

24) $\frac{5x-7}{3x-1}=2+\frac{2x}{3x-1}$ 25) $\frac{3x}{x^2-4x}-\frac{7}{x}=\frac{3}{x-4}$

F: Simplify:

26) $(x-2)\left(x^2-4x+3\right)-(5x+1)$ 27) $\left(3x^3-x+5\right)-\left(3x^2-9x-8x^3+2\right)$

28) $(x+2)\left(x^3-x^2+x\right)$

G: Divide and simplify:

29) $\dfrac{x^2-7x+12}{2x^2-7x-4} \div \dfrac{x-5}{2x^2-5x-3}$

30) $\dfrac{3x^2+x-2}{4x^2-6x} \div \dfrac{6x^2-x-2}{2x^2-x-3}$

31) $\dfrac{x^2-9}{3x^2-11x+6} \div \dfrac{2x^2+5x-3}{x^2-3x}$

H: Story problems:

32) The length of a rectangular garden is 5 feet longer than three times the width. If the owner plans to use 238 feet of fencing to enclose the garden, find the dimensions of the garden.

33) The width of a rectangle is 2/3 of its length. When each dimension is increased by 5 feet, the perimeter is 130 feet, Find the dimensions of the original rectangle.

34) A motorboat can maintain a constant speed of 16 miles per hour in still water. The boat makes a trip upstream to a certain point in 2/5 of an hour and then travels back to the starting point downstream in ¼ of an hour. Find the rate of the current to the nearest tenth.

35) Two cars which were 219 miles apart at 3 PM are traveling toward each other from opposite directions. The blue car started at 3 PM and is traveling at a constant rate of 60 mph. The red care started at 3:30 PM and is traveling at a constant rate of 66 mph. At what time did the cars meet?

36) A pharmacist must prepare 28 ml of a solution which contains 3.5% of an active ingredient. However, the pharmacist only has solutions of 2% active ingredient and 7% active ingredient in stock. How many ml of each type of solution should be used to prepare the 28 ml solutions with 3.5% active ingredient?

37) How many gallons of a solution containing 80% alcohol should be added to 6 gallons of a solution containing 25% alcohol to obtain a solution containing 30% alcohol?

38) Jack can paint his house in 10 hours. If Jill helps they can paint the house together in 3 hr 20 min. How long would it take Jill to paint the house by herself?

39) Pete must borrow money to pay $5537 for tuition at Purdue. Whatever amount he borrows, 2% in fees in deducted, so that Pete actually receives 2% less than the amount he borrows. How much must Pete borrow to pay his tuition?

Answers to review, exam 1:

1) $\dfrac{x^{10}}{9y^4}$ 2) $\dfrac{xy^3}{2}$ 3) $\dfrac{1}{8x^5y^4}$ 4) $\dfrac{y^2}{25x^4}$ 5) $r=\dfrac{t^2}{m+mt}$

6) $b=\dfrac{2A-hc}{h}$ 7) $h=\dfrac{2A-2L^2}{b}$ 8) $k=\dfrac{\Pr}{A-P}$

9) $x^2y(4x^2+9)(2x+3)(2x-3)$ 10) $3(x+2)(x^2-5)$ 11) $2x(4x+5)(2x-1)$

12) $(2x-3)(x^2+4)(x+2)(x-2)$ 13) $3(x-2)(5x+4)$ 14) $x^2(1+7x)(1-7x)$

15) $2(x-1)(2x+21)$ 16) $3(9y^2+4)(3y+2)(3y-2)$ 17) $a(1-b)$

18) $\dfrac{b}{a}$ 19) $\dfrac{b+a^2}{ab^2}$ 20) $x+2$ 21) -1

22) no solution $\left(-\dfrac{5}{3}\text{ makes the den }0\right)$ 23) $\dfrac{11}{8}$ 24) $-\dfrac{5}{3}$

25) $\varnothing$ (4 makes den = 0) 26) x^3-6x^2+6x-7 27) $11x^3-3x^2+8x+3$

28) $x^4+x^3-x^2+2x$ 29) $\dfrac{(x-3)^2}{x-5}$ 30) $\dfrac{(x+1)^2}{2x(2x+1)}$

31) $\dfrac{x(x-3)}{(3x-2)(2x-1)}$ 32) 28.5 x90.5 33) 33 x 22 34) 3.7 mph

35) 1.5 hours: meet at 5:00 36) 19.6 ml of 2%, 8.4% of 7%

37) .6 gal 38) 5 hours 39) $5650

Review for exam 2

A: Solve:

1) $6x^2 + x - 12 = 0$

2) $5x^2 + 13x = 6$

3) $\frac{5}{x^2} - \frac{10}{x} + 2 = 0$

4) $x^2 + 3x + 6 = 0$

5) $x^4 + 5x^2 - 36 = 0$

6) $|2x+1| \le 3$

7) $\sqrt{5x-9} = x+1$

8) $-3x \ge 0$

9) $-10 \le 3x + 5 < \frac{1}{4}$

B: Write in the form $a + bi$

10) $(7-6i) - (-11-3i)$

11) $(4+9i)(4-9i)$

12) $(3-\sqrt{-25})(2+i)$

13) $\frac{5}{2-7i}$

14) $\frac{2+3i}{3-2i}$

C: Coordinate plane problems:

15) Find the distance, midpoint, and slope of the line segment with the endpoints A(-2, -5) and B(4, 6)

16) Given one endpoint A(2, -5) and the midpoint M(4, 9), find the other endpoint B.

17) Find the point with coordinates of the form (2a, a) that is in the 3^{rd} quadrant and is a distance of 5 away from the point P(1, 3).

18) Write the equation of the line that satisfies the given conditions: Leave the equation in standard form, where a>0

- through (2, 3) and (-1, 5)
- through (2, 3) perpendicular to the line with the equation $y = \frac{2}{3}x + 4$
- through the line with an x-intercept of 5 and a y-intercept of -2

D: Circles: (sometimes on test 3: check to see where this review belongs)

19) Given the equation of a circle $(x+4)^2+(y-5)^2=60$, find the center and radius of the circle.

20) Given the equation of a circle $x^2+y^2-10x+18=0$, find the center and radius of the circle.

21) Given the diameter of the endpoints of a circle as (-2, 4) and (7, 3), write the equation of the circle in standard form.

E: Graphs:

22) Graph the curve with the given equation:

- $x=\sqrt{y+1}$
- $y=\sqrt{9-x^2}$

F: Story problems – look over old tests

Answers for review exam 2:

1) $\frac{4}{3}$, $-\frac{3}{2}$

2) $\frac{2}{5}, \quad -3$

3) $\frac{5 \pm \sqrt{15}}{2}$

4) $\frac{-3 \pm i\sqrt{15}}{2}$

5) $\pm 3i, \quad \pm 2$

6) $[-2,1]$

7) $\frac{3 \pm i\sqrt{31}}{2}$

8) $(-\infty, 0]$

9) $\left(-5, -\frac{19}{12}\right]$

10) $18-3i$

11) 97

12) $11-7i$

13) $\frac{10+35i}{53}$

14) i

15) $D = \sqrt{157}, \quad MP = \left(1, \frac{1}{2}\right)$

16) (6, 23)

17) (-2, -1)

18) $a) y = -\frac{2}{3}x + \frac{13}{3} \quad b) y = -\frac{3}{2}x + 6$

18c) $y = \frac{2}{5}x - 2$

19) C(-4, 5), r = $\sqrt{60}$

20) C (5, 0), r = $\sqrt{7}$

21) $\left(x - \frac{5}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{41}{2}$

22) some points are: (0, 1) (1, 0) (2, 3)

23) some points are: (3, 0) (0, 3) (-3, 0)

Review for exam 3

1) Find the minimum value of the function $f(x) = x^2 - 2x - 8$

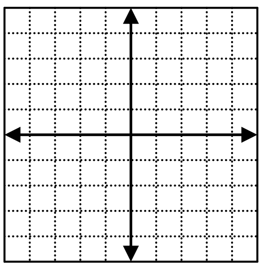
2) The graph of the function $f(x) = 5x^2 + 20x + 17$ is a parabola.
   a) Put this is standard form   b) Find the minimum or maximum of this function.

3) The graph of $f(x) = -2x^2 + 4x - 1$ is a parabola.
   a) Write this in standard form   b) f(x – 2) is also a function. Find its vertex.

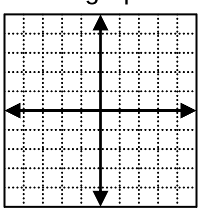
4) Given $f(x) = -3x^2 + 6x - 7$, find the vertex and the maximum or minimum value of the function.

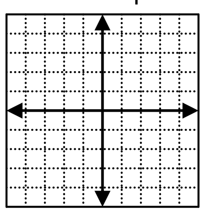
5) Find the standard equation of the parabola that has a vertex V(-3, -2) and an x-intercept 1.

6) Given the graph of $y = f(x)$ below, graph $g(x) = f(x+3) - 2$



7) Given the graphs below, which best expresses the relationship between the functions g and f?





$A) g(x) = f(x+2)+1$
$B) g(x) = f(x-2)-1$
$C) g(x) = f(x-1)-2$
$D) g(x) = f(x-2)+1$
$E) g(x) = f(x+1)+2$

8) Let $f(x) = \dfrac{5}{3x+1}$. Find the inverse function of f. Simplify your answer.

9) Find the inverse function $f^{-1}(x)$, of the function $f(x) = \dfrac{2}{x-3}$

10) If $f(x) = \sqrt{x-3} + 2,\ \text{find}\, f^{-1}(x)$

11) Find the inverse function of $f(x) = \dfrac{2}{x+7}$

12) Find the inverse function of f: $f(x) = 4x + 3$

13) y is directly proportional to the square of *w* and inversely proportional to the product of *x* and *z*. a) Express *y* in terms of *w, x,* and *z*, and a constant of proportionality, *k.*
b) If *w* = 3, *x* = 4, and *z* = 5, then y = 18. Find y when w = -2, x = 5, and z = 10

14) y is directly proportional to the product of x and the cube of w and inversely proportional to the square root of v.
a) Express y in terms of x, w, v and a constant of proportionality k
b) If y = 3 when x = 4, w = -1, and v = 25, find the value of k from part a.

15) z is directly proportional to the product of x and the square of y, and inversely proportional to the cube root of w. Express z in terms of x, y, w, and a constant of proportionality, k.

16) Many rivers, particularly many in Brazil, have the property that the actual length of the river, L, is directly proportional to the straight distance, D, from source to mouth.
a) Write L as a function of D
b) The Amazon River has a straight distance from source to mouth of 1241 miles and an actual length of 3900 miles. Use this information to find the straight distance from source to mouth of the Xingu River (also in Brazil) if its actual length is 2670 miles. Give your answer to the nearest mile.

17) y varies directly as the square root of x and inversely as the cube of z. If y = 1 when x = 4 and z = 3, find y when x = 9 and z = 3.

18) Find the domain of the function $h(x) = \dfrac{\sqrt{x-3}}{x-4}$

19) The domain of the function $f(x) = \dfrac{x+3}{2x^2+x-6}$ is all real numbers <u>except</u>:

20) Given y = *f(x)*, with domain [-5. 10] and the range [-12, -3], fine the domain and range of y =f(x – 3)

21) Find the domain of *f* in interval notation: $f(x) = \dfrac{4\sqrt{x+5}}{x^2-49}$

22) The slope, m, of the perpendicular bisector of $\overline{AB}$ where A (-3, 4) and B (5, -2) is:

23) Find the equation of the line through the point (-1, 3) and perpendicular to the line 5x – 3y = -2

24) Find the equation of the line perpendicular to segment AB and through point A.
Given A (5, -2) and B (7, 6). Give your answer in general form.

25) Find all values of x, in interval notation, such that f(x) > 0 and all x such that f(x) < 0.
$f(x) = (x-3)^2(x+2)$

26) If $f(x) = x^2+5$, find $\dfrac{f(a+h)-f(a)}{h}$

27) Let $f(x) = 3x^2-4$ and $g(x) = x+5$. Find and simplify:
a) (f – g)(-2) b) (g o f)(x)

28) Given $g(x) = 6x$ and $f(x) = 2x^2 - 4$, find $(fog)(3)$

29) If $f(x) = 2x^2 + 1$ and $g(x) = 3 - x$, find (g o f)(x)

30) For two function f and g, you are given: f(2) = 3, f(-2) = 7, g(2) = -2, g (7) = 2.
Find each of the following: a) (f + g)(2) b) (fg)(2) c) f[g(7)] d) g[g(7)]

31) Let $f(x) = x^2 + 2$ and $g(x) = x - 7$. Find each of the following:
a) (fg)(5) b) (f – g)(5) c) g[f(-1)] d) f[f(-1)]

32) Given $f(x) = 3x - 6$ and $g(x) = 2x^2$, find g[f(x)]

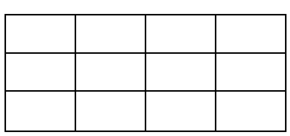
Solve the following systems of equations algebraically. Do not graph:

33) $\begin{cases} 4x - 9y = 9 \\ xy = 1 \end{cases}$ 34) $\begin{cases} x^2 - 3y = 6 \\ x^2 + y^2 = 10 \end{cases}$ 35) $\begin{cases} 2x + 3y = 10 \\ 3x + 5y = 17 \end{cases}$

36) What is the maximum number of points any circle and any line can have in common? Draw a sketch to support your answer.

37) Find <u>all</u> points that the circle $x^2 + y^2 = 10$ and the line $x + 2y = 1$ have in common.

38) A rectangular field is going to be enclosed by a fence and the field will also be divided into twelve rectangular plots by placing three fences parallel to the shorter sides and two fences parallel to the longer sides (see figure). If there are 1600 total feet of fencing available, express the total enclosed area, A, of the rectangular field as a function of the length of the longer side, x. You needn’t simplify your expression



39) A farmer has 200 meters of fencing to enclose a rectangular region adjacent to a river. No fencing is needed along the river. (See sketch)
a) Express w as a function of l
b) Express the area enclosed as a function of l

40) The perimeter of a rectangle is 23 meters and its area is 30 square meters. Find its dimensions. Name the variable, set up equations and solve.

41) Twelve gallons of a 25% acid solution is to be obtained by mixing a 10% solutions with a 50% solution. How may gallons of each type of solution should be used?

42) Brazil nuts are \$4.50 per pound and cashews are \$3.00 per pound. How many pounds of each type of nuts is needed to be mixed to obtain 96 pounds of mixed nuts worth \$4.20 per pound?

43) Since the beginning of the month, a local reservoir has been losing water at a constant rate. On the 12^{th} of the month, the reservoir held 200 million gallons of water. On the 21^{st}, it held only 164 million gallons of water. Assume that the amount of water, W, is linearly related to time, t, since the beginning of the month. Let t = 1 correspond to the first day of the month)
a) Express the amount of water in the reservoir (in millions of gallons), W, as a function of time, t.
b) How much water was in the reservoir o the 8^{th} of the month?

44) A painting is worth \$500 in 1982 and appreciates to \$1025 by 1992. Assuming a linear growth, express the painting's value, V, in terms of time t, in years, where t = 0 corresponds to the year 1980.

45) A truck and a car left from the same point. The truck began traveling east at 2:00 PM at the constant speed of 30 mph. Two hours later, the car started traveling north at the constant speed of 60 mph. Let t represent the time (in hours) <u>the car has been traveling.</u> Express the distance in miles, d, between the car and truck as a function of time, t. DO NOT SIMPLIFY YOUR EXPRESSION.

46) Graph the piecewise-defined function. Label at least three points on the graph:

$$f(x) = \begin{cases} 3x & \text{if} \quad x < 1 \\ 2 - x & \text{if} \quad x \geq 1 \end{cases}$$

Answers to practice for exam 3:

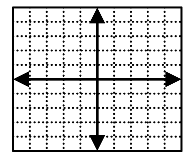
1) -9

2) a) $5(x+2)^2-3$ b) $\min=-3$

3) $-2(x-1)^2+1, \quad V=(3,4)$

4) V (1, -4) max: -4

5) $y=\frac{1}{8}(x+3)^2-2$

6) 

7) A

89) $y=\frac{5-x}{3x}$

9) $y=\frac{2+3x}{x}$

10) $(x-2)^2+3$

11) $y=\frac{2-7x}{x}$

12) $y=\frac{x-3}{4}$

13) a) $y=\frac{kw^2}{xz}$ b) $y=\frac{16}{5}$

14) a) $y=\frac{kxw^3}{\sqrt{v}}$, b) $k=-\frac{15}{4}$

15) $z=\frac{kxy^2}{\sqrt[3]{w}}$

16) a) $L=KD$, b) 849

17) $\frac{4}{9}$

18) [3, 4] $\cup(4,\infty)$

19) $x\neq\frac{3}{2}, \quad -2$

20) D: [-8, 7] R: [3, 12]

21) $\left[-\frac{5}{4},7\right)\cup(7,\infty)$

22) $\frac{4}{3}$

23) 3x + 5y = 12

24) x + 4y = -3

25) $\begin{cases} f(x)>0 \Rightarrow (-2,3)\cup(3,\infty) \\ f(x)<0 \Rightarrow (-\infty,-2) \end{cases}$

26) $2a+h$

27) 5 b) $3x^2+1$

28) 644

29) $-2x^2+2$

30) a) 1 b) -6 c) 3 d) -2

31) a) -54 b) 29 c) -4 d) 11

32) $18x^2-72x+72$

33) $\left(-\frac{3}{4},-\frac{4}{3}\right)$ and $\left(3,\frac{1}{3}\right)$

34) (3, 1) and (-3, 1)

35) (-1, 4)

36) 2

37) $\left(-\frac{13}{5},\frac{9}{5}\right)$ and $(3,\text{-}1)$

38) $A = x\left(\frac{1600-4x}{5}\right)$

39) a) $w = \frac{200-l}{2}$, b) $A = l\left(\frac{200-l}{2}\right)$

40) 7.5 by 4

41) x = 7.4 y = 4.5

42) B = 19.2 C= 76.8

43) a) $w = -4t + 248$ b) 216

44) $V = 52.5t + 395$

45) $\sqrt{(60t)^2 + (30t+60)^2}$

46)