MA 23100 - Practice Exam 2

1. Find $f'(4)$ when $f(x) = 4(x^2+1)(\sqrt{x}-3)$.

   A. $f'(4) = 272$

   B. $f'(4) = 36$

   C. $f'(4) = -15$

   D. $f'(4) = 96$

   E. $f'(4) = \dfrac{-111}{4}$

2. For $f(x) = 2x^4 + 10x^3 + 17x + 12$, what is $f^{(4)}(0)$?

   A. 0

   B. 48

   C. 24

   D. 12

   E. 10

3. Find the derivative of $y = (2x - 3)\sqrt{4x + 1}$.

   A. $\dfrac{dy}{dx} = \dfrac{12x - 4}{\sqrt{4x + 1}}$

   B. $\dfrac{dy}{dx} = 8\sqrt{4x + 1}$

   C. $\dfrac{dy}{dx} = \dfrac{18x + 1}{2\sqrt{4x + 1}}$

   D. $\dfrac{dy}{dx} = \dfrac{16x - 10}{\sqrt{4x + 1}}$

   E. $\dfrac{dy}{dx} = 2\sqrt{4x + 1}$

4. The ozone level (in parts per billion) in a metropolitan area is modeled by

$$P(t) = 60 + 15t - t^2$$

   where t is time in hours and $t = 0$ corresponds to 8:00 am. The rate of change of the ozone level at 1:00 pm is:

   A. 60 ppb/hr

   B. 15 ppb/hr

   C. 13 ppb/hr

   D. 7 ppb/hr

   E. 5 ppb/hr

5. Find the equation of the line tangent to $f(x) = \sin^2 x$ at $x = \dfrac{\pi}{4}$.

   A. $y = \sqrt{2}x - \dfrac{2 - \sqrt{2}\pi}{4}$

   B. $y = x - \dfrac{\pi}{4}$

   C. $y = \sqrt{x} + \dfrac{1 - \pi}{4}$

   D. $y = x + \dfrac{2\sqrt{2} - \pi}{4}$

   E. $y = x + \dfrac{2 - \pi}{4}$

6. If $r = \dfrac{1}{\sec\theta - \tan\theta}$, then $\left.\dfrac{\mathrm{d}r}{\mathrm{d}\theta}\right|_{\theta=5\pi} =$

   A. -2

   B. 0

   C. -1

   D. 1

   E. 2

7. Let $f(u) = \dfrac{u^2-1}{u^2+1}$ and $g(x) = \cos\dfrac{x}{2}$. Find $(f \circ g)'(\dfrac{\pi}{2})$.

   A. $-\dfrac{4}{9}$
   B. $\dfrac{8}{9}$
   C. -3
   D. 0
   E. $-\dfrac{8}{9}$

8. Consider the function $f'(x) = \dfrac{x^2}{x^2+7}$. Choose the correct statement about the critical points of f.

   A. f has no critical points.
   B. f has only one critical point.
   C. f has exactly two critical points.
   D. f has exactly three critical points.
   E. f has exactly four critical points.

9. Consider the function $f(x) = x^4 - 6x^2 + 9$. Choose the correct statement about the relative extrema of f.

   A. f has one relative minimum and no relative maximum.
   B. f has two relative maxima and no relative minimum.
   C. f has two relative minima and one relative maximum.
   D. f has one relative maximum and one relative minimum.
   E. f has two relative maxima and one relative minimum.

10. Which one of the following functions has $(1, 5)$ as a point of inflection?

   A. $f(x) = (x-1)^2 + 5$
   B. $f(x) = (x-1)^3 + 1$
   C. $f(x) = (x+1)^2 + 1$
   D. $f(x) = (x+1)^3 - 3$
   E. $f(x) = (x-1)^3 + 5$

11. Among the following statements, how many are true?

    I. If $f'(x) > 0$ over an interval, then $f(x)$ is increasing over that interval.

    II. If $f'(c) = 0$ and $f''(c) > 0$ then $(c, f(c))$ is a relative maximum.

    III. If $f''(x) < 0$ over an interval, then $f(x)$ is decreasing over that interval.

    IV. The point at which the concavity of a curve changes has to be either a relative minimum or a relative maximum of the function.

    A. None of the statements is true.

    B. Exactly one statement is true.

    C. Exactly two statements are true.

    D. Exactly three statements are true.

    E. All of the statements are true.

12. For a function f, we know that $f''(x) > 0$ when $\frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$ and $f''(x) < 0$ elsewhere on the real line. The function's relative extrema are at $x = -1, x = 0, x = 1$. Classify them.

    A. f has a relative maximum at $x = 0$ and relative minima at $x = -1, x = 1$.

    B. f has a relative minimum at $x = 1$ and relative maxima at $x = -1, x = 0$.

    C. f has a relative maximum at $x = -1$ and relative minima at $x = 0, x = 1$.

    D. f has a relative minimum at $x = 0$ and relative maxima at $x = -1, x = 1$.

    E. f has a relative minimum at $x = -1$ and relative maxima at $x = 0, x = 1$.