MA 23100 - Practice Exam 3

1. If $f(x) = x + 1 + \dfrac{4}{x+3}$, then $f'(x) = 1 - \dfrac{4}{(x+3)^2}$ and $f''(x) = \dfrac{8}{(x+3)^3}$.
   Where do the relative extrema of $f(x)$ occur?

   A. $x = -6,\ x = 0$
   B. $x = -5,\ x = -3,\ x = -1$
   C. $x = -6,\ x = -3,\ x = 0$
   D. $x = -5,\ x = -1$
   E. $x = -3,\ x = -1$

2. Given that $f(x) = 2x^3 - 5x^2 - 4x + 291$ has exactly two critical points at $x = -\frac{1}{3}$ and $x = 2$, what are the largest and smallest values of $f(x)$, rounded to the nearest hundredth, attained on the interval $[0, 3]$?

   A. largest value: 291, smallest value: 279
   B. largest value: 291, smallest value: 288
   C. largest value: 288, smallest value: 279
   D. largest value: 292, smallest value: 288
   E. largest value: 288, smallest value: 274

3. Write the following equation in logarithmic form: $e^{(2^x)} = y$.

   A. $x = \log_2(\ln(y))$
   B. $x = \log_{e^2}(y)$
   C. $x = \ln(\log_2(y))$
   D. $x = \log_{2e}(y)$
   E. $x = \ln^2(y)$

4. Find $\dfrac{dy}{dx}$ if $y = x^2e^x + \cos(e^{2x})$.

   A. $e^x\left(x^2 + 2x - 2e^2\sin\left(e^{2x}\right)\right)$
   B. $\left(x^2 + 2x\right)e^x + \left(2\cos\left(e^{2x}\right) - \sin\left(e^{2x}\right)\right)e^{2x}$
   C. $\left(x^2 + 2x\right)e^x - 2e^{2x}\sin\left(e^{2x}\right)$
   D. $\left(x^2 + 2x\right)e^x + 2e^{2x}\sin\left(e^{2x}\right)$
   E. $e^x\left(x^2 + 2x + 2e^2\sin\left(e^{2x}\right)\right)$

5. Find $\left.\dfrac{dy}{dx}\right|_{x=0}$ if $y = \dfrac{e^{x^2+7x}}{x-1}$.

   A. 6
   B. -7
   C. 7
   D. -2
   E. -8

6. Find $\dfrac{dy}{dx}$ if $\sin x + \sin y = y \cos x$.

   A. $\dfrac{-y \sin x}{\cos x - \cos y}$
   B. $\dfrac{y \sin x + \cos x}{\cos x - \cos y}$
   C. $\dfrac{\cos x}{\cos x - \cos y}$
   D. $\dfrac{y \sin x + \cos y + \cos x}{\cos x}$
   E. $\dfrac{y \sin x + \cos y + 1}{\cos y}$

7. A cylindrical tank with radius 3 m is being filled with water at a rate of $4\,\mathrm{m}^3/\mathrm{min}$. The volume of a cylinder is $V = \pi r^2 h$, where r is the radius of the circular ends, and h is the height of the cylinder. How fast is the height of the water increasing?

   A. $\frac{3}{2}$ m/min
   B. $\frac{1}{\pi}$ m/min
   C. $\frac{4}{6\pi}$ m/min
   D. $\frac{4}{9\pi}$ m/min
   E. $\frac{4}{\pi}$ m/min

8. Approximate $\sqrt[5]{33}$ **using linearization**.
   Hint: $2^5 = 32$.

   A. 2.009
   B. 2.014
   C. 1.9875
   D. 2.0125
   E. 2.01235

9. The population of a city is 39000 and is increasing at a rate of 3.75% per year. How long will it take for the population to exceed 50000?

   A. 6.75 years
   B. 2.35 years
   C. 1.06 years
   D. 4.25 years
   E. 5.89 years

10. If $f(x) = \dfrac{x^5 + 7x^2 - 1}{x^2 + 3x + 2}$, find the number of correct statements.

   I. $\lim_{x \to -\infty} f(x) = -\infty$.
   II. $f(x)$ has a horizontal asymptote.
   III. $f(x)$ has two vertical asymptotes.
   IV. $f(x)$ has a slant/oblique asymptote.

   A. There are zero correct statements
   B. There is only one correct statement
   C. There are only two correct statements
   D. There are only three correct statements
   E. All statements are correct

11. A farmer wants to create a rectangular grazing field for his cows. He wants the field to boarder a large creek so his cows can drink from it, which means he only needs fencing for three sides. If the farmer purchased 500 ft of fencing material, what is the largest possible area for the grazing field?

    A. $15625\,\text{ft}^2$
    B. $31250\,\text{ft}^2$
    C. $42500\,\text{ft}^2$
    D. $55500\,\text{ft}^2$
    E. $62500\,\text{ft}^2$

12. A box of volume $50\,\text{in}^3$ whose base length l is 3 times the base width w is to be built. The material used to build the top and bottom cost $10\,¢/\text{in}^2$ and the material used to build the sides cost $6\,¢/\text{in}^2$. What is the cost of the box (in dollars) in terms of the width w?

    A. $C(w) = 0.2w^2 + \frac{4}{w}$
    B. $C(w) = 0.1w^2 + \frac{8}{w^2}$
    C. $C(w) = 0.3w^2 + \frac{3}{w}$
    D. $C(w) = 0.6w^2 + \frac{8}{w^2}$
    E. $C(w) = 0.6w^2 + \frac{8}{w}$