<u>MA 23200 - Practice Exam 3</u>

1. Compute AB-C where A= $\begin{bmatrix} 1 & 1 \\ 0 & 3 \\ 5 & 2 \end{bmatrix}$, B= $\begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}$, and C= $\begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 6 & 4 \end{bmatrix}$

A. $\begin{bmatrix} 0 & 1 \\ -9 & -2 \\ 5 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 1 \\ 9 & -2 \\ -5 & -1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & -1 \\ -9 & -2 \\ -5 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 1 \\ -9 & 2 \\ 5 & -1 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 1 \\ -9 & -2 \\ -5 & -1 \end{bmatrix}$

2. Solve the initial value problem

$$\begin{cases} f''(x) = e^{-\frac{1}{3}x} \\ f(0) = 1,\ f'(0) = 2 \end{cases}$$

A. $-\frac{1}{9}e^{-\frac{1}{3}x} + 5x - 8$

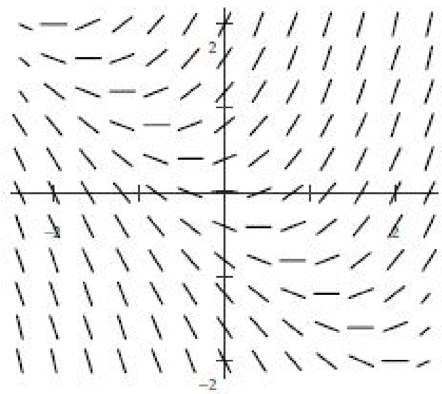
B. $9e^{-\frac{1}{3}x} + 5x - 8$

C. $\frac{1}{9}e^{-\frac{1}{3}x} + 5x - 8$

D. $\frac{1}{9}e^{-\frac{1}{3}x} + 5x + 8$

E. $-9e^{-\frac{1}{3}x} + 5x - 8$

3. Which differential equation has the direction field shown below?



A. $\frac{dy}{dx} = 1 + x$

B. $\frac{dy}{dx} = x^2$

C. $\frac{dy}{dx} = \frac{x}{y}$

D. $\frac{dy}{dx} = \ln y$

E. $\frac{dy}{dx} = x + y$

4. Use Euler's method with $\Delta x = .1$ to approximate $y(0.2)$ if $y'(x) = x + 2y$ and $y(0) = 2$.
Euler's method: $y_{n+1} = y_n + f(x_n, y_n)\Delta x$, where $y_n \approx y(x_n)$.

A. 2.40

B. 3.09

C. 2.89

D. 2.68

E. 3.49

5. Which best describes the stability of the equilibrium values of the autonomous differential equation

$$y' = -y^3 - y^2 + 6y.$$

A. 2 stable and 1 unstable

B. 1 stable, 1 semistable, and 1 unstable

C. 2 unstable and 1 semistable

D. 2 semistable and 1 unstable

E. 1 stable and 2 unstable

6. Find the interval on which a unique solution exists for the initial value problem

$$(x+3)y' - \frac{1}{x+4}y = \frac{e^{4x}}{x-2}; \qquad y(1) = 3.$$

A. $(-4, 2)$

B. $(-\infty, 2)$

C. $(-3, \infty)$

D. $(-3, 2)$

E. $(-4, \infty)$

7. Find the integrating factor $G(x)$ of the differential equation

$$(\cos x + 2)y' - (\sin x)y = 2.$$

A. $\cos x$

B. $\sin x + 2$

C. $\cos x - 2$

D. $\sin x$

E. $\cos x + 2$

8. Solve the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(y^2+1)\sin x}{y^3+y}; \qquad y(0) = 2.$$

A. $y(x) = -\sqrt{-2\cos x + 6}$

B. $y(x) = \sqrt{-2\cos x - 6}$

C. $y(x) = \sqrt{-2\cos x + 3}$

D. $y(x) = -\sqrt{-2\cos x + 3}$

E. $y(x) = \sqrt{-2\cos x + 6}$

9. The population of a city is increasing at a rate inversely proportional to the population at time t, where t is the number of years after 1993. If the population in 1993 ($t = 0$) was 2 thousand, and in 1994 was 4 thousand, what was the population in 1998?

   A. 6 thousand

   B. 7 thousand

   C. 8 thousand

   D. 9 thousand

   E. 10 thousand

10. Given the initial value problem

$$y' - 2axy = -bx; \qquad y(0) = \frac{b}{a},$$

where a and b are nonzero constants. The integrating factor G(x) of the differential equation $y' - 2axy = -bx$ is $G(x) = e^{-ax^2}$. Solve for $y(x)$.

   A. $y(x) = \frac{b}{2a} + \frac{b}{2a}e^{ax^2}$

   B. $y(x) = -\frac{b}{2a} + \frac{b}{2a}e^{-ax^2}$

   C. $y(x) = -\frac{b}{a} + \frac{b}{a}e^{ax^2}$

   D. $y(x) = \frac{b}{a} + \frac{b}{a}e^{ax^2}$

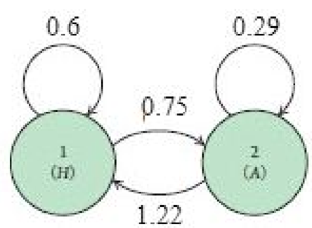
   E. $y(x) = \frac{b}{2a} + \frac{b}{a}e^{-ax^2}$

11. How many of the following differential equations are separable, linear or autonomous?

$$y' = ye^{-3x} + e^{-3x} \qquad 2(y^2+1)y' = \ln y \qquad y' + 3y = 1.$$

A. 1 separable, 1 linear, and 1 autonomous
B. 1 separable, 2 linear, and 2 autonomous
C. 3 separable, 2 linear, and 1 autonomous
D. 3 separable, 2 linear, and 2 autonomous
E. 2 separable, 2 linear, and 2 autonomous

12. The Leslie diagram for a bird population is shown below



In Year 1, the hatchling population is 33 and the adult population is 18. Let a and b be the hatchling and adult population respectively in Year 3. Choose the correct statement.

A. $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1.22 & 0.75 \\ 0.6 & 0.29 \end{bmatrix} \left(\begin{bmatrix} 1.22 & 0.75 \\ 0.6 & 0.29 \end{bmatrix} \begin{bmatrix} 33 \\ 18 \end{bmatrix} \right)$

B. $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.75 & 0.29 \\ 0.6 & 1.22 \end{bmatrix} \left(\begin{bmatrix} 0.75 & 0.29 \\ 0.6 & 1.22 \end{bmatrix} \begin{bmatrix} 33 \\ 18 \end{bmatrix} \right)$

C. $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.6 & 1.22 \\ 0.75 & 0.29 \end{bmatrix} \left(\begin{bmatrix} 0.6 & 1.22 \\ 0.75 & 0.29 \end{bmatrix} \begin{bmatrix} 33 \\ 18 \end{bmatrix} \right)$

D. $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.29 & 0.6 \\ 0.75 & 1.22 \end{bmatrix} \left(\begin{bmatrix} 0.29 & 0.6 \\ 0.75 & 1.22 \end{bmatrix} \begin{bmatrix} 33 \\ 18 \end{bmatrix} \right)$

E. $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1.22 & 0.6 \\ 0.75 & 0.29 \end{bmatrix} \left(\begin{bmatrix} 1.22 & 0.6 \\ 0.75 & 0.29 \end{bmatrix} \begin{bmatrix} 33 \\ 18 \end{bmatrix} \right)$