

MA 22000 Lesson 22
(2nd half of textbook, Section 4.5)
Derivatives of Natural Logarithmic Functions

Derivative of $\ln x$:

$$\frac{d}{dx} (\ln x) = \frac{1}{x}$$

The derivative of $\ln x$ is the reciprocal of x .

*There is a justification for this rule on page 237 of the textbook. We will accept it as true without proof.

There is also a rule on page 237 of the text for taking derivatives of logarithmic expressions to a base other than base e . You may study some of these types of problems, but none are on homework or quizzes/tests.

This rule is as follows: Let a be any logarithmic base other than e , then

$$\frac{d}{dx} [\log_a x] = \frac{1}{(\ln a)x}$$

**CHAIN RULE FOR DERIVATIVE OF A LOGARITHMIC
FUNCTION**

Let u = a function of the independent variable x . Then the derivative of u :

$$\frac{d}{dx} (\ln u) = \frac{1}{u} (u')$$

The derivative of u (a function of x) is the reciprocal of u times the derivative of u .

For the next few problems, *assume the value of the variable is positive.*

Ex 1: Find the derivative of $y = \ln(12x)$

(Chain rule must be used where $u = 12x$.)

NOTE:

If c is a positive constant,

$$\frac{d}{dx} \ln(cx) = \frac{1}{cx} (c) = \frac{1}{x}$$

Ex 2: Find $D_x[\ln(x^2 - 4x)]$

Ex 3: Find the derivative of f , where $f(x) = \ln(3x^3 - x)^{3/2}$.

Ex 4: If $y = \frac{3x+2}{2\ln x}$, find y' .

Ex 5: Find the derivative of $g(x) = (\ln 2)[\ln(12x)]$
(Note: natural log of 2 is a constant.)

Ex 6: $y = e^x (\ln x)$

Ex 7: $f(x) = \sqrt{e^x - \ln(3x)}$
(Use chain rule)

Ex 8:

The total revenue received from the sale of x items is given by

$R(x) = 20 \ln(x+1)$ and the total cost to produce x items is given by

$$C(x) = \frac{2x}{3}.$$

- a) Find the marginal revenue.
- b) Find the profit function.
- c) Find the marginal revenue when $x = 40$.
- d) Find the marginal profit when $x = 40$.

Ex 9: Suppose the demand function for x units of a certain product is

$$p = D(x) = 70 + \frac{10}{\ln x}, \quad x > 1 \text{ and } p \text{ is in dollars.}$$

- a) Find the marginal revenue.
- b) Approximate the revenue from one more unit when 10 units are sold.