MA 22400 FORMULAS

## CONSUMERS' AND PRODUCERS' SURPLUS

$$\text{CS} = \int_0^{q_0} D(q)dq - p_o q_0 \qquad \text{PS} = p_o q_0 - \int_0^{q_0} S(q)dq$$

## TRAPEZOIDAL RULE

$$\int_a^b f(x)dx \equiv \frac{\Delta x}{2}\Big[f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_n) + f(x_{n+1})\Big],$$

where $a = x_1, x_2, x_3, \ldots, x_{n+1} = b$ subdivides $[a, b]$ into $n$ equal subintervals of length $\Delta x = \dfrac{b-a}{n}$.

## THE SECOND DERIVATIVE TEST

Suppose $f$ is a function of two variables $x$ and $y$, and that all the second-order partial derivatives are continuous. Let

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

and suppose $(a, b)$ is a critical point of $f$.

1. If $D(a, b) < 0$, then $f$ has a saddle point at $(a, b)$,
2. If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then $f$ has a relative maximum at $(a, b)$.
3. If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then $f$ has a relative minimum at $(a, b)$.
4. If $D(a, b) = 0$, the test is inconclusive.

## LAGRANGE EQUATIONS

For the function $f(x, y)$ subject to the constraint $g(x, y) = k$, the Lagrange equations are

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad g(x, y) = k$$

## LEAST-SQUARES LINE

The equation of the least-squares line for the $n$ points $(x_1,y_1)$, $(x_2,y_2)$, ... , $(x_n,y_n)$, is
$y = mx + b$, where

$$m = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} \qquad b = \frac{\sum x^2 \sum y - \sum x \sum xy}{n\sum x^2 - \left(\sum x\right)^2}$$

## GEOMETRIC SERIES

If $0 < |r| < 1$, then

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

## TAYLOR SERIES

The Taylor series of $f(x)$ about $x = a$ is the power series

$$\sum_{n=0}^{\infty} a_n(x-a)^n \quad \text{where} \quad a_n = \frac{f^{(n)}(a)}{n!}$$

Examples:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } -\infty < x < \infty; \qquad \ln x = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(x-1)^n, \text{ for } 0 < x \le 2$$

## VOLUME & SURFACE AREA

**Right Circular Cylinder**

$V = \pi r^2 h$

$$SA = \begin{cases} 2\pi r^2 + 2\pi rh \\ \pi r^2 + 2\pi rh \end{cases}$$

**Sphere**

$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

**Right Circular Cone**

$V = \frac{1}{3}\pi r^2 h$

$SA = \pi r\sqrt{r^2 + h^2} + \pi r^2$