Formulas

1. Linearization for a function $f(x, y)$ of two variables, the linearization at the point (a, b) is given by:

$$f(x, y) \approx f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b).$$

2. D-Test to find the relative maximum and minimum values of f:

   (1) Find f_x, f_y, f_{xx}, f_{yy} and f_{xy}.
   (2) Solve $f_x(x, y) = 0$ and $f_y(x, y) = 0$.
   (3) Evaluate $D = f_{xx}f_{yy} - [f_{xy}]^2$ at each point (a, b) found in Step 2.
      (a) If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f has a relative maximum at (a, b).
      (b) If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f has a relative minimum at (a, b).
      (c) If $D(a, b) < 0$, then f has a saddle point at (a, b).
      (a) If $D(a, b) = 0$, then the test is inconclusive. You will have to do something else to determine what is happening at that point.

3. Method of Least Squares.
   The line of least squares regression for the n points (c_1, d_1), (c_2, d_2),...,(c_n, d_n) is given by:

$$y - \bar{y} = m(x - \bar{x})$$

   where,

$$\bar{x} = \frac{\sum_{i=1}^{n} c_i}{n}, \qquad \bar{y} = \frac{\sum_{i=1}^{n} d_i}{n}, \qquad m = \frac{\sum_{i=1}^{n}(c_i - \bar{x})(d_i - \bar{y})}{\sum_{i=1}^{n}(c_i - \bar{x})^2}.$$