Formulas

1. The **linearization** for a function $f(x,y)$ of two variables at the point (a,b) is given by:

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b).$$

2. **D-Test** to find the relative maximum and minimum values of f:

   (1) Find f_x, f_y, f_{xx}, f_{yy} and f_{xy}.

   (2) Solve $f_x(x,y) = 0$ and $f_y(x,y) = 0$.

   (3) Evaluate $D = f_{xx}f_{yy} - [f_{xy}]^2$ at each point (a,b) found in Step 2.

   - (a) If $D(a,b) > 0$ and $f_{xx}(a,b) < 0$, then f has a relative maximum at (a,b).
   - (b) If $D(a,b) > 0$ and $f_{xx}(a,b) > 0$, then f has a relative minimum at (a,b).
   - (c) If $D(a,b) < 0$, then f has a saddle point at (a,b).
   - (a) If $D(a,b) = 0$, then the test is inconclusive. You will have to do something else to determine what is happening at that point.

3. **Method of Least Squares**.

The line of least squares regression for the n points (c_1,d_1), (c_2,d_2),...,(c_n,d_n) is given by:

$$y - \bar{y} = m(x - \bar{x})$$

where,

$$\bar{x} = \frac{\sum_{i=1}^{n} c_i}{n}, \qquad \bar{y} = \frac{\sum_{i=1}^{n} d_i}{n}, \qquad m = \frac{\sum_{i=1}^{n}(c_i - \bar{x})(d_i - \bar{y})}{\sum_{i=1}^{n}(c_i - \bar{x})^2}.$$

4. **Euler's Method**.

The approximation of the solution to $y' = f(x,y)$, $y(x_0) = y_0$ using Euler's method with increments of Δx is:

$$y_{n+1} = y_n + f(x_n, y_n)\Delta x$$

where $x_{n+1} = x_n + \Delta x$ and $y_n \approx y(x_n)$.