# MA 232 Practice Exam 3

1. $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 1 \\ 0 & 3 \\ 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 4 \\ 1 & 2 \\ 3 & 1 \end{bmatrix}$, and $v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Compute $A(B+C)v$.

2. The population of a certain species of birds is divided into two groups: hatchlings (H) and adults (A). The Leslie matrix for this population is $A = \begin{bmatrix} 0.2 & 1.1 \\ 0.8 & 0.6 \end{bmatrix}$. Draw the Leslie diagram. If there are 30 hatchlings and 10 adults in Year 1, how many adults will there be in Year 2?

3. $f''(x) = \sin(\frac{x}{3})$, $f'(3\pi) = 3$, and $f(\frac{\pi}{2}) = 5$. Find $f(\pi)$.

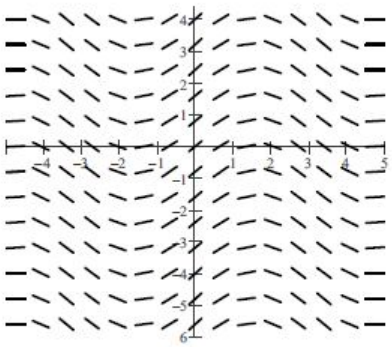
4. Consider the initial value problem

$$y' + \frac{1}{x^2 - 1}y = \frac{\sin x}{2x - 9} \qquad\qquad y(2) = 0.$$

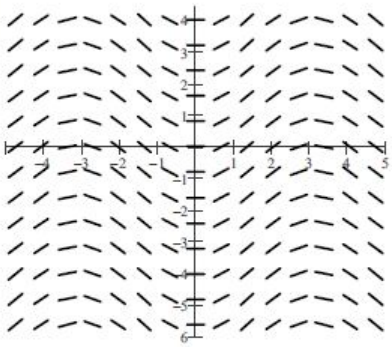
What is the largest interval on which a unique solution will exist. Do not attempt to find the solution.

5. Which choice represents the slope field for $\frac{dy}{dx} = \cos x$?

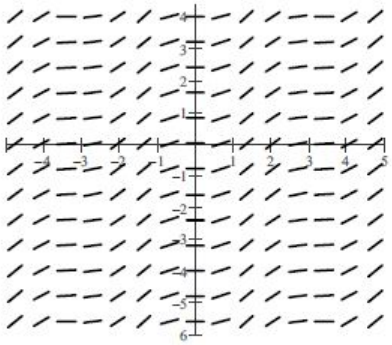
(A)



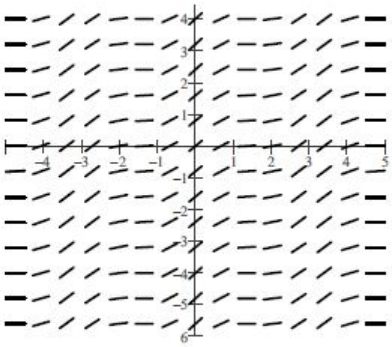
(B)



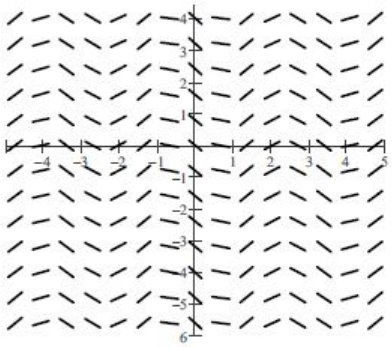
(C)



(D)



(E)



6. Find the solution to the differential equation

$$\frac{1}{x}y' = \frac{e^y + 2}{e^y}.$$

Keep your answer implicitly if necessary.

7. Find the solution to the initial value problem

$$xy' + 2y = 5x^3 \qquad\qquad y(1) = \frac{5}{4}.$$

8. Consider the autonomous differential equation

$$y' = y^4 - 7y^3 + 10y^2.$$

Determine the equilibrium value(s), and classify each one as stable, unstable or semistable.

9. Let $y$ be the solution to the initial value problem

$$y' = x(y+1) \qquad\qquad y(1) = 2.$$

Use Euler's method with $\Delta x = 0.2$ to approximate $y(1.4)$.
Euler's method: $y_{n+1} = y_n + f(x_n, y_n)\Delta x$, where $y_n \approx y(x_n)$.

10. A certain species has a population of 625 at $t = 0$. This population is increasing at a rate proportional to the square root number of the population at time $t$. If at $t = 1$, the population is 1600, what would the population be when $t = 2$?

11. How many of the following differential equations are linear, separable or autonomous?

$y' = \sin y + \frac{1}{y}$

$y' + x^2y^2 = x^2$

$(x^2 + 1)y' - xy = 1$

12. A mussel is placed into polluted water containing polychlorinated biphenyls (PCBs). Let $Q(t)$ be the concentration of PCB in the mussel (in micrograms of PCB per gram of tissue) after $t$ days. For low concentrations of pollution, the mussel absorbs PCBs at the rate of $N_0$ micrograms of PCB per gram of tissue per day for some constant $N_0$. Also, the elimination rate of PCBs from the mussel is $0.2Q$ micrograms per gram of tissue per day. $Q(t)$ satisfies the differential equation

$$Q' = -0.2Q + N_0.$$

Assume $Q(0) = Q_0$, where $Q_0$ is some positive constant. Find $Q(1)$.