MA 232 Practice Exam 2

1. Given $f(x, y) = e^y + 2xy^3 + 3x^2y$, find f_y.

   Answer: $e^y + 6xy^2 + 3x^2$.

2. Given $f(x, y) = \sin(x^2y)$, find $f_x(1, \frac{\pi}{3})$.

   Answer: $\dfrac{\pi}{3}$.

3. $z = \ln(x^2 + 2y^2)$. Find $f_{xy}(2, 1)$.

Answer: $-\dfrac{4}{9}$.

4. Given $f(x, y) = \dfrac{x^2}{y}$, use linearization at $(4, 2)$ to estimate $f(4.02, 1.97)$.

Answer: 8.20.

5. The first order partial derivatives of $f(x, y) = 10x^3 + 3y^2 - 30xy + 90x + 6y + 19$ are

$$f_x = 30x^2 - 30y + 90 \qquad f_y = 6y - 30x + 6.$$

Find the relative extrema and saddle points of $f(x, y)$ if they exist.

Answer: relative minimum at $(4, 19)$, saddle point at $(1, 4)$, no relative maximum.

6. A flat plate is located on a coordinate plane. The temperature of the plate, in degrees Fahrenheit, at point (x, y) is given by

$$T(x, y) = x^2 + y^2 - 9x - 3y.$$

Find the maximum temperature if it exists. If not, state there is no maximum temperature.

Answer: There is no maximum temperature.

7. Find the slope of the least-squares regression line for the following set of points: $(1, 2)$, $(2, 4)$, $(4, 4)$, and $(5, 2)$.
Answer: 0.

8. Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \int_0^{\sec y} \tan y \mathrm{d}x \mathrm{d}y$.

Answer: $\sqrt{2} - 1$.

9. Evaluate $\int_1^2 \int_0^x 3y^2 \ln(x^2) \mathrm{d}y \mathrm{d}x$.

Answer: $4 \ln 4 - \frac{15}{8}$.

10. Let $f(x, y) = x+y$, and G be the region bounded by the curves of $y = x$ and $y = x^2$. Integrate $f(x, y)$ over the region G. Leave your answer in a double integral format.

Answer: $\int_0^1 \int_{x^2}^x (x + y) \mathrm{d}y \mathrm{d}x$.

11. Evaluate $\int\int_G e^{x+y}\mathrm{d}y\mathrm{d}x$, where G is the region bounded by the x-axis, $y = \ln x$, and $x = \ln 4$.

Answer: convergent; $4\ln 4 - 8 + e$.

12. $y'(x) = \sin x \cos^2 x$. Find $y(0)$ if $y(\frac{\pi}{2}) = 0$.

Answer: $-\frac{1}{3}$.