FINAL EXAM MATH 26100 FALL 2011

Name ______________________________

Student ID ______________________________

Recitation Instructor ______________________________

Recitation Section and Time ______________________________

INSTRUCTIONS:

1. This exam contains 25 problems each worth 8 points.
2. Please supply all information requested above on the scantron.
3. Work only in the space provided, or on the backside of the pages. You must show your work.
4. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
5. No books, notes, or calculators, please.

Mark **TEST 01** on your scantron!

KEY: ACBDD BACED EACED EEAAB CBCDB

1. Assume $f(x, y, z) = y\sqrt{xz}$. Evaluate the gradient of f at $(1, -2, 1)$.

A. $\langle -1, 1, -1 \rangle$

B. $\langle -2, 1, -2 \rangle$

C. $\langle 1, -1, 1 \rangle$

D. $\langle 1, -2, 1 \rangle$

E. $\langle -1/2, 1, -1/2 \rangle$

2. Find the mass of the region above the plane $z = 0$ and between the surfaces $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ when the density function is given by $\rho(x, y, z) = 6(x^2 + y^2 + z^2)^{3/2}$.

A. $2\pi(4^6 - 1)$

B. $4\pi(2^6 - 1)$

C. $2\pi(2^6 - 1)$

D. $4\pi(4^6 - 1))$

E. $10\pi/6$

3. If C is the curve that travels along $x^2 + y^2 = 4$ counter-clockwise from the point $(2, 0)$ to the point $(0, 2)$, then evaluate

$$\int_C 2xy \; ds$$

A. 2π

B. 8

C. $\pi/2$

D. 2

E. 6

4. Let C be the curve that traverses the triangle with vertices $(0, 0)$, $(3, 0)$, and $(0, 2)$ counter-clockwise once. Compute

$$\int_C P dx + Q dy$$

where $P(x, y) = xy$ and $Q(x, y) = x^2 + 2$.

A. 0

B. 8/3

C. 7/3

D. 3

E. 3/2

5. A particle starts at the `origin` with initial velocity $\mathbf{i}-\mathbf{j}+3\,\mathbf{k}$ and acceleration $\mathbf{a}(t) = 6t\,\mathbf{i}-12t^2\,\mathbf{j}-6t\,\mathbf{k}$. At what point is the particle at $t = 2$?

A. $(10, 18, -4)$
B. $(10, -12, -4)$
C. $(10, 18, -2)$
D. $(10, -18, -2)$
E. $(10, 12, -6)$

6. Let P be the point of intersection of the curves $\mathbf{r}_1(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}$ and $\mathbf{r}_2(t) = (1+t)\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$. What is the angle of intersection for these curves at the point P ?

A. $\dfrac{2\pi}{3}$

B. $\dfrac{\pi}{2}$

C. $\dfrac{\pi}{3}$

D. $\dfrac{\pi}{4}$

E. $\dfrac{\pi}{6}$

7. What is the area of the triangle with vertices $(2, 0, 0)$, $(0, 4, 0)$, $(0, 0, 6)$?

A. 14
B. 18
C. 22
D. 26
E. 28

8. Let $\mathbf{u} = \langle 1, -2, 3 \rangle$ and $\mathbf{v} = \langle 2, 1, 1 \rangle$. If $\mathbf{w} = \langle a, b, c \rangle$ is perpendicular to $\mathbf{u} \times \mathbf{v}$ which of the following hold?

A. $b = 2a + c$

B. $c = a + b$

C. $a = b + c$

D. $b = a + c$

E. $a = b + 2c$

9. Find the minimum value of $f(x, y) = x^2 - xy + y^2 + 1$ on the circle $x^2 + y^2 = 1$.

A. 3/4

B. 1

C. 2

D. 1/2

E. 3/2

10. If $f(x, y) = x^3 + 6xy - y^3$, which statement is true.

A. f has a local min. and a local max.
B. f has a saddle point and a local max.
C. f has two saddle points
D. $f(2, -2)$ is a local min.
E. $f(0, 0)$ is a local min.

11. Let R be the triangle with vertices $(0,0),(1,0)$ and $(1,2)$. Evaluate the integral

$$\iint_R (e^{x^2} - 2y)\, dA$$

A. $(e-1)/2$

B. $2e-1$

C. $(3e-5)/3$

D. $(2e-1)/2$

E. $(3e-7)/3$

12. At what value does the tangent plane for the surface $z = 4x^2 - y^2 + 2y$ at the point $(-1,2,4)$ intersect with the z axis?

A. 0
B. 4
C. -4
D. 8
E. 12

13. A solid in the first octant is bounded by the surfaces $x^2 + z^2 = 16$, $2y = x$, $y = 0$ and $z = 0$. Which integral gives the volume of the solid?

A. $\int_0^2 \int_0^{x/2} \sqrt{16 - x^2}\ dzdx$

B. $\int_0^4 \int_0^{2x} \sqrt{16 - y^2}\ dydx$

C. $\int_0^4 \int_0^{x/2} \sqrt{16 - x^2}\ dydx$

D. $\int_0^2 \int_0^{2x} \sqrt{16 - x^2}\ dydx$

E. $\int_0^4 \int_0^{2y} \sqrt{16 - z^2}\ dxdy$

14. Let R be the region in the first quadrant that is bounded by $y = 0$ and the circle $(x-1)^2 + y^2 = 1$. Then

$$\iint_R y\ dA =$$

A. 1/3

B. 4/3

C. $-4/3$

D. $-2/3$

E. 2/3

15. Let E be the bounded solid in the first octant bounded by the planes.

$$y = x,\ x = 0,\ z = 0 \text{ and } y + z = 1.$$

Evaluate

$$\iiint_E (1 + y)\ dV$$

A. 2

B. 3/2

C. 1/2

D. 1/4

E. 3/4

16. Find the limit if it exists:

$$\lim_{(x,y)\to(0,0)} \frac{y^2 \cos y}{x^2 + 9y^2}$$

A. 1/10

B. 1/9

C. 1

D. 0

E. the limit does not exist

17. Use implicit differentiation to compute $\frac{\partial z}{\partial y}$ when $x = e, y = 2, z = 1$ assuming $xyz - e^z - e = 0$.

A. $1/e$

B. 1

C. e

D. $1/3e$

E. -1

18. Use the linear approximation of the function

$$f(x, y) = \sqrt{x^2 + y^2}$$

at $(3, 4)$ to approximate $f(3.1, 4.2)$.

A. 5.22

B. 5.28

C. 5.3

D. 5.32

E. 5.34

19. Let $z = \cos(x + y^2)$, $x = f(t)$, $y = g(t)$, where $f(0) = 2$, $g(0) = 1$, $f'(0) = 3$, $g'(0) = -4$. Find $\frac{dz}{dt}$ at $t = 0$.

A. $5 \sin 3$

B. 0

C. $\sin 3$

D. $2 \sin 3$

E. $-2 \sin 3$

20. The maximum rate of change of $f(x, y) = \ln(x + y)$ at the point $(1, 0)$ is:

A. $\sqrt{5}$

B. $\sqrt{2}$

C. 5

D. 2

E. 3

21. Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = xz\mathbf{i} + xz^3\mathbf{j} + z^2\mathbf{k}$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$, and the planes $z = 1$, and $z = -1$.

A. $-\pi$

B. -2π

C. 0

D. π

E. 3π

22. Given $\mathbf{F}(x, y, z) = -e^z y\mathbf{i} + x\mathbf{j} + x^2\mathbf{k}$. Use Stoke's Theorem to calculate $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$ where S is the hemisphere $x^2 + y^2 + z^2 = 4$ for $z \geq 0$ oriented by the upward normal.

A. 0

B. 8π

C. 4π

D. 6π

E. 2π

23. Calculate the surface integral $\iint_S z dS$ when S is the part of the cone $z = \sqrt{x^2+y^2}$ such that $1 \le x^2 + y^2 \le 4$.

A. $2\sqrt{2}\ \pi$

B. $\sqrt{2}\ \pi$

C. $14\sqrt{2}\ \pi/3$

D. $2\pi/3$

E. $8\sqrt{2}\ \pi/3$

24. Find the downward pointing unit normal to the surface $z = x^3 + xy^2$ at $(1, 2, 5)$.

A. $\langle -7, -4, 1\rangle$

B. $\langle 7, 4, -1\rangle$

C. $\langle \frac{-7}{\sqrt{65}}, \frac{-4}{\sqrt{65}}, \frac{-1}{\sqrt{65}}\rangle$

D. $\langle \frac{7}{\sqrt{66}}, \frac{4}{\sqrt{66}}, \frac{-1}{\sqrt{66}}\rangle$

E. $\langle \frac{-7}{\sqrt{65}}, \frac{-4}{\sqrt{65}}, \frac{1}{\sqrt{65}}\rangle$

25. Evaluate

$$\iiint_E yx^2 \, dV$$

Where E is the solid obtained by rotating the triangle with vertices $(0,0,0)$, $(0,0,1)$, and $(1,0,1)$ counter-clockwise about the z-axis from $\theta = 0$ to $\theta = \pi/2$. *hint: Use cylindrical coordinates.*

A. $\pi/30$

B. $1/90$

C. $1/15$

D. 2π

E. $\pi/2$