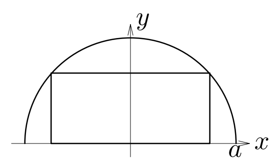
MA 16100 FINAL EXAM PRACTICE PROBLEMS

1. $\lim\limits_{x\to 1} \frac{x^2-1}{x^2-x} =$ A. -1 B. 0 C. 1 D. 2 E. Does not exist
2. If $y = (x^2+1)\tan x$, then $\frac{dy}{dx} =$ A. $2x\tan x + (x^2+1)\sec^2 x$ B. $2x\sec^2 x$ C. $2x\tan x + (x^2+1)\tan x$ D. $2x\tan x + 2x\sec^2 x$ E. $2x\tan x$
3. If $h(x) = \begin{cases} x^2+a, & \text{for } x < -1 \\ x^3-8 & \text{for } x \geq -1 \end{cases}$ determine all values of a so that h is continuous for all values of x. A. $a=-1$ B. $a=-8$ C. $a=-9$ D. $a=-10$ E. There are no values of a.
4. Evaluate $\lim\limits_{x\to 0^+} x\cos(\frac{1}{x})$. (Hint: $-1 \leq \cos(\frac{1}{x}) \leq 1$ for all $x \neq 0$.) A. 0 B. 1 C. -1 D. $\frac{\pi}{2}$ E. Does not exist
5. If $f(x) = \frac{1}{x+3}$, then $\lim\limits_{x\to 1} \frac{f(x)-f(1)}{x-1} =$ A. $\frac{1}{4}$ B. $\frac{1}{16}$ C. $-\frac{1}{16}$ D. $-\frac{1}{4}$ E. Does not exist
6. The equation $x^3 - x - 5 = 0$ has one root for x between -2 and 2. The root is in the interval: A. $(-2,-1)$ B. $(-1,0)$ C. $(0,1)$ D. $(1,2)$ E. $(-1,1)$
7. If $f(x) = \frac{1-x}{1+x}$, then $f'(1) =$ A. -1 B. $-\frac{1}{2}$ C. 0 D. $\frac{1}{2}$ E. 1
8. If $y = \ln(1-x^2) + \sin^2 x$, then $\frac{dy}{dx} =$ A. $\frac{1}{1-x^2} + \cos^2 x$ B. $\frac{1}{1-x^2} + 2\sin x\cos x$ C. $\frac{1}{1-x^2} + 2\sin x$ D. $\frac{-2x}{1-x^2} + \cos^2 x$ E. $\frac{-2x}{1-x^2} + 2\sin x\cos x$
9. Find $f''(x)$ if $f(x) = \frac{1-x}{1+x}$ A. $\frac{4}{(1+x)^3}$ B. $\frac{-4}{(1+x)^3}$ C. $-\frac{4x}{(1+x)^3} + \frac{2}{(1+x)^2}$ D. $\frac{2(1+x)^2-2x(1+x)}{(1+x)^4}$ E. -1
10. Assume that y is defined implicitly as a differentiable function of x by the equation $xy^2 - x^2 + y + 5 = 0$. Find $\frac{dy}{dx}$ at $(-2,1)$. A. 9 B. $\frac{-5}{3}$ C. 1 D. 2 E. $\frac{5}{3}$
11. Find the maximum and minimum values of the function $f(x) = 3x^2+6x-10$ on the interval $-2 \leq x \leq 2$. A. max is 14, min is -10. B. max is -10, min is -13 C. max is 14, min is -13 D. no max, min is -10 E. max is 14, no min.
12. For a differentiable function $f(x)$ it is known that $f(3) = 5$ and $f'(3) = -2$. Use a linear approximation to get the approximate value of $f(3.02)$. A. 6.02 B. 5.02 C. 5.04 D. 3 E. 4.96.
13. Water is withdrawn from a conical reservoir, 8 feet in diameter and 10 feet deep (vertex down) at the constant rate of 5 ft^3/min. How fast is the water level falling when the depth of the water in the reservoir is 5 ft? ($V = \frac{1}{3}\pi r^2 h$). A. $\frac{15}{16\pi}$ ft/min B. $\sqrt{\frac{3}{\pi}}$ ft/min C. $\frac{2}{\pi}$ ft/min D. $5\sqrt[3]{3/4\pi}$ ft/min E. $\frac{5}{4\pi}$ ft/min.
14. A rectangle is inscribed in the upper half of the circle $x^2+y^2=a^2$ as shown at right. Calculate the area of the largest such rectangle. A. $\frac{a^2}{2}$ B. $3a\sqrt{2}$ C. $2a^2$ D. $4a^2$ E. a^2.

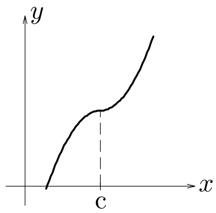


15. Given that $f(x)$ is differentiable for all x, $f(2) = 4$, and $f(7) = 10$, then the Mean Value Theorem states that there is a number c such that A. $2 < c < 7$ and $f'(c) = \frac{6}{5}$ B. $2 < c < 7$ and $f'(c) = \frac{5}{6}$ C. $4 < c < 10$ and $f'(c) = \frac{6}{5}$ D. $2 < c < 7$ and $f'(c) = 0$ E. $4 < c < 10$ and $f'(c) = 0$.
16. Suppose that the mass of a radioactive substance decays from 18 gms to 2 gms in 2 days. How long will it take for 12 gms of this substance to decay to 4 gms? A. $\frac{\ln 3}{\ln 2}$ days B. 1 day C. $\frac{\ln 2}{\ln 3}$ days D. 2 days E. $(\ln 3)^2$ days
17. Which of the following is/are true about the function $g(x) = 4x^3 - 3x^4$? (1) g is decreasing for $x > 1$. (2) g has a relative extreme value at $(0,0)$. (3) the graph of g is concave up for all $x < 0$. A. (1), (2) and (3) B. only (2) C. only (1) D. (1) and (2) E. (1) and (3).
18. Find where the function $f(x) = 2/\sqrt{1+x^2}$ is increasing A. all x B. no x C. $x < 0$ D. $x > 0$ $x = 0$.
19. Let f be a function whose <u>derivative</u>, f', is given by $f'(x) = (x-1)^2(x+2)(x-5)$. The function has A. a relative maximum at $x=-2$ and a relative minimum at $x=5$. B. a relative maximum at $x=5$ and a relative minimum at $x=-2$. C. relative maxima at $x=1$, $x=-2$ and a relative minimum at $x=5$. D. a relative maximum at $x=5$ and relative minima at $x=1$, $x=-2$ E. a relative maximum at $x=1$ and relative minima at $x=-2$, $x=5$.
20. Find $\frac{d}{dx}\int_1^{2x} \sqrt{t^2+1}\,dt$ at $x=\sqrt{2}$. A. 6 B. 3 C. $\sqrt{2}$ D. $\sqrt{4x^2+1}$ E. $\frac{1}{2\sqrt{3}}$.
21. $\int_3^4 x\sqrt{25-x^2}\,dx =$ A. 0 B. -37 C. $\frac{37}{3}$ D. $-\frac{74}{3}$ E. $\frac{7}{12}$

22. $\lim\limits_{x\to\infty} \frac{x^2+2x}{3x^2+4} =$ A. 1 B. 3/7 C. 1/4 D. 0 E. 1/3.

23. $\lim\limits_{x\to 0} \frac{2x-\sin^{-1}x}{2x+\tan^{-1}x} =$ A. 1/2 B. 2 C. 1/3 D. 1 E. 0.

24. Suppose that a function f has the following properties:
$f''(x) > 0$ for $x < c$, $f'(c) = 0$, and $f'(x) < 0$ for $x > c$.
Which of the following could be the graph of f?

A.


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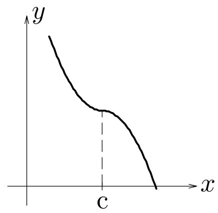

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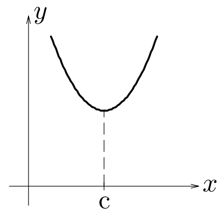

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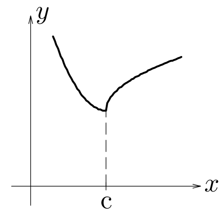

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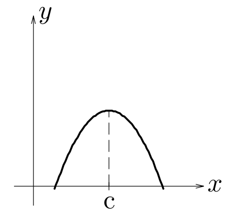


25. Let R be the region between the graph of $y = \frac{1}{x}$ and the x-axis, from $x = a$ to $x = b$ $(0 < a < b)$. If the vertical line $x = c$ cuts R into two parts of equal area, then $c =$ A. $\sqrt{ab}$ B. $\frac{a+b}{2}$ C. $\frac{\ln a + \ln b}{2}$ D. $\ln\left(\frac{a+b}{2}\right)$ E. $\ln\left(\frac{b-a}{2}\right)$

26. Find the area of the region between the graph of $y = \frac{1}{1+x^2}$ and the x–axis, from $x = -\sqrt{3}$ to $x = 1$. A. $\frac{\pi}{2}$ B. $\frac{3\pi}{4}$ C. $\frac{15\pi}{12}$ D. $\frac{\pi}{3}$ E. $\frac{7\pi}{12}$

27. $\frac{d}{dx}\left(e^{2x}\ln\sqrt{1+x}\right) =$ A. $e^{2x}\ln(1+x) + \frac{e^{2x}}{2(1+x)}$ B. $\frac{e^{2x}}{\sqrt{1+x}} + 2e^{2x}\ln\sqrt{1+x}$ C. $\frac{1}{2}e^{2x}\ln(1+x) + \frac{e^{2x}}{2(1+x)}$ D. $\frac{2e^{2x}}{\sqrt{1+x}}$ E. $\frac{e^{2x}}{1+x}$

28. $\frac{d}{dx}x^{\sin x} =$ A. $(\cos x)x^{\sin x}$ B. $(\sin x)x^{\sin x - 1}$ C. $x^{\cos x}$ D. $x^{\sin x}[\frac{\sin x}{x} + (\cos x)\ln x]$ E. $(\ln x)x^{\sin x}$

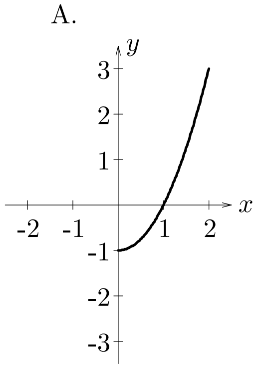
29. $\frac{d}{dx}\tan^{-1}e^{3x} =$ A. $\frac{1}{1+e^{3x}}$ B. $\frac{e^{3x}}{1+e^{3x}}$ C. $\frac{3e^{3x}}{1+e^{6x}}$ D. $\frac{3e^{3x}}{1+e^{9x^2}}$ E. $\frac{3e^{3x}}{\sqrt{1-e^{6x}}}$

30. $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}dx =$ A. $\frac{\pi}{2}$ B. $\frac{\pi}{6}$ C. $\sin^{-1}\sqrt{3}$ D. $\frac{\pi}{3}$ E. 1

31. $\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx =$ A. $\frac{7}{2}$ B. $\frac{10}{3}$ C. $\frac{11}{4}\tan^{-1}3$ D. 3 E. 4

32. $\int_0^1 \frac{e^x}{1+e^x}dx =$ A. $\ln\frac{1+e}{2}$ B. $\ln(1+e)$ C. $\frac{1}{2}$ D. $1-\ln 2$ E. e

33. If $f(x) = x^2 - 1$, $0 \le x \le 2$, then the graph of $y = f^{-1}(x)$ is

A.


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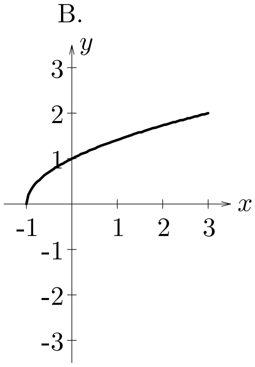

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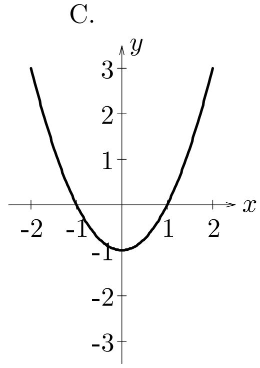

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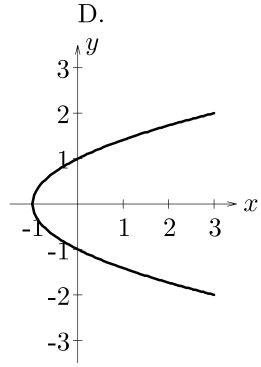

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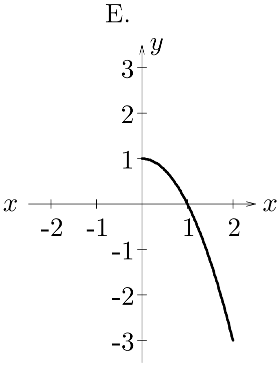

Answers: 1.D, 2.A, 3.D, 4.A, 5.C, 6.D, 7.B, 8.E, 9.A, 10.E, 11.C, 12.E, 13.E, 14.E, 15.A, 16.B, 17.C, 18.C, 19.A, 20.A, 21.C, 22.E, 23.C, 24.B, 25.A, 26.E, 27.A, 28.D, 29.C, 30.D, 31.B, 32.A, 33.B