MA 15910 Lesson 29 Notes

If f is a function on an interval. Let c be a number in that interval. Then…

(a) $f(c)$ is the absolute maximum of f on that interval if $f(x) \leq f(c)$ for every x in the interval.

(In other words, every other y or function value is lower or less than $f(c)$.)

(b) $f(c)$ is the absolute minimum of f on that interval if $f(x) \geq f(c)$ for every x in the interval.

(In other words, every other y or function value is larger or greater than $f(c)$.)

Note: Sometimes the textbook refers to an absolute maximum or absolute minimum as an absolute extremum.

Also of note: Just as a relative maximum or a relative minimum was the y-value or function value, so it is with absolute extrema. The x-value is the ***location*** of an absolute or relative maximum/minimum.

Look at Figure 2 below. (This is the same figure 2 that is on page 305 of the 2^{nd} half of your textbook.)

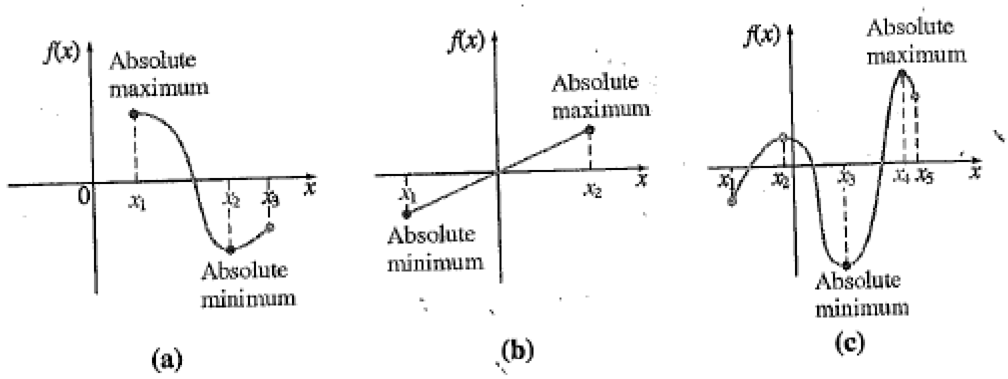
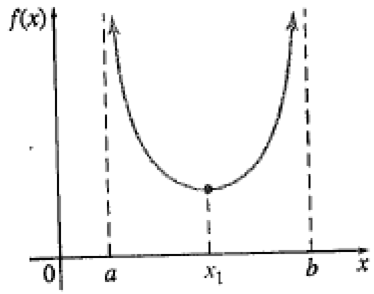


FIGURE 2

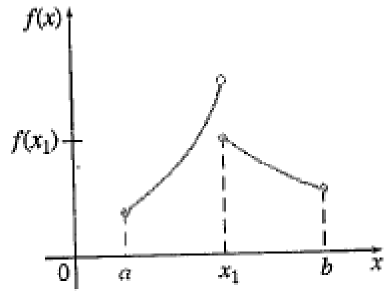
In (a), there is an absolute maximum at x_1 and an absolute minimum at x_2. $f(x_1)$ is a greater value than all of function values (y-values) in the interval $[x_1, x_3]$. In (b), there is an absolute minimum at x_1 and an absolute maximum at x_2 in the closed interval, $[x_1, x_2]$. In (c), for the interval $[x_1, x_5]$, the absolute minimum is $f(x_3)$ at x_3 and the absolute maximum is $f(x_4)$ at x_4.

Although a function can have <u>only one absolute minimum value and only one absolute maximum value (in a specified interval)</u>, it can have more than one location or points where these values occur. For any function in a closed interval [*a*, *b*], there will be an absolute maximum and an absolute minimum on that interval.

If a function is continuous on an open interval, there may or may not be an absolute maximum or an absolute minimum. **∞ or -∞ cannot be absolute extrema, only exact real numbers can be absolute extrema.** Examine figure 3(a) below. There is an absolute minimum at x_1, but there is no absolute maximum value, since the greatest function value goes toward infinity. Also, a function that has a 'break' or 'gap' at a value of *x* may or may not have an absolute minimum or maximum. Look at figure 3(b) below. In the closed interval [*a*, *b*], there is an absolute minimum at *x* = a, but there is no absolute maximum value. The actual function value at x_1 is lower than the open circle above. There is not an absolute maximum at x_1.



(a)



(b)

To find absolute extrema of a function *f* on a <u>closed interval [*a, b*]</u>, follow these steps.

1. Find all critical values of *f* in the open interval (*a*, *b*).
2. Evaluate *f* (find function values) for those critical values in (*a*, *b*) **and** the endpoints *a* and *b* of the closed interval [*a*, *b*].
3. The largest value found is the absolute maximum for *f* on [*a*, *b*}, and the smallest value found is the absolute minimum for *f* on [*a*, *b*].

<u>Ex. 1:</u> Find the absolute minimum and absolute maximum values of $f(x) = x^2 - 8x + 10$ on the interval $[0, 7]$. Where do these values occur?

Ex 2: Find the absolute extrema of the function $f(x) = x^3 - 3x^2$ on $[-1,1]$. Where are the points where these absolute extrema occur?

Ex 3: Find the absolute maximum and absolute minimum of $h(x) = \frac{1}{3-x}$ on $[0, \frac{11}{4}]$.

Ex 4: Find the absolute extrema of $f(x) = (x-1)^{2/3}$ on [-7, 2] and where they occur.

Ex 5:
A retailer has determined the cost *C* for ordering and storing *x* units of a product to be modeled by the cost function, $C(x) = 3x + \frac{30000}{x}$, $1 \le x \le 200$. (The delivery truck can bring at most 200 units per order.) Find the size of the order that will minimize the cost.

To find any possible absolute extrema on an <u>open interval</u>, follow these steps.

1. Find all critical values in the open interval. Evaluate the function values at these critical values.
2. Find the limits as the endpoints are approached (or as x approaches ∞ or $-\infty$). If a limit is infinity or negative infinity, these cannot be considered for absolute extrema.
3. The greatest function value is the absolute maximum and the least is the absolute minimum.

Find the absolute extrema, if they exist, as well as all values of x where they occur.

<u>Ex 6:</u> $f(x) = \dfrac{x}{x^2+1}$

<u>Ex 7:</u> $g(x) = x\ln x$

Ex. 8:

A company has found that its weekly profit from the sale of *x* units of an auto part is given by $P(x) = -0.02x^3 + 600x - 20000$. Production limits the number of units that can be made per week to no more than 150, and a contract requires that at least 50 units be made each week. Find the maximum possible weekly profit that the company can make.

Ex 9:

A fast-food restaurant has determined that the monthly demand for its hamburgers is given by $price = p = \frac{60000 - x}{20000}$ and its cost function is given by $C = 5000 + 0.56x$. The greatest number of hamburgers that the restaurant can make and sell in a month is 50,000. Find the production level (number of hamburgers) that will maximize profit.

Ex 10:

The number of salmon swimming upstream is approximated by $S(x) = -x^3 + 3x^2 + 360x + 5000, \quad 6 \le x \le 20$ where x represents the temperature of the water in degrees Celsius. Find the water temperature that produces the maximum number of salmon swimming upstream.