MA 16010 - Practice Exam 2

1. If $h(t) = \sin(3t) + \cos(3t)$, find $h^{(3)}(t)$.

   A. $\sin(3t) - \cos(3t)$
   B. $\sin(3t) + \cos(3t)$
   C. $27\sin(3t) - 27\cos(3t)$
   D. $27\sin(3t) + 27\cos(3t)$
   E. $-27\sin(3t) + 27\cos(3t)$

2. Given $f(x) = \dfrac{2(3-x^2)}{\sqrt{3x^2+1}}$. Find $f'(1)$.

   A. $-\frac{7}{2}$
   B. $-\frac{9}{4}$
   C. $-\frac{1}{2}$
   D. $-\frac{13}{6}$
   E. $-\frac{3}{4}$

3. A spherical balloon is inflated with gas at a rate of 5 cubic centimeters per minute. How fast is the radius of the balloon changing at the instant the radius is 4 centimeters? The volume V of a sphere with a radius r is $V = \frac{4}{3}\pi r^3$.

   A. $\frac{5}{64\pi}$ centimeters per minute
   B. $\frac{25}{4\pi}$ centimeters per minute
   C. $\frac{5}{16\pi}$ centimeters per minute
   D. $\frac{256\pi}{3}$ centimeters per minute
   E. $\frac{5}{4\pi}$ centimeters per minute

4. A toy rocket is launched from a platform on earth and flies straight up into the air. Its height after launch is given by:

$$s(t) = t^3 + 3t^2 + 4t + 16,$$

   where s is measured in meters, and t is in seconds. Find the velocity when the acceleration is 18 m/s^2.

   A. 2 m/s
   B. 44 m/s
   C. 16 m/s
   D. 28 m/s
   E. 13 m/s

5. According to a joint study conducted by Oxnard's Environmental Management Department and a state government agency, the concentration of CO in the air due to automobile exhaust t yr from now is given by

$$C(t) = 10(0.2t^2 + 4t + 64)^{\frac{2}{3}}$$

parts per billion. Find the rate at which the level of CO is changing 20 years from now. Round your answer to the nearest integer.

A. 9 parts per billion per year

B. 11 parts per billion per year

C. 13 parts per billion per year

D. 19 parts per billion per year

E. 25 parts per billion per year

6. Find $\dfrac{dy}{dx}$ by implicit differentiation.

$$\ln(xy) + 2x = e^y$$

A. $\dfrac{dy}{dx} = \dfrac{-2 - y}{x - e^y}$

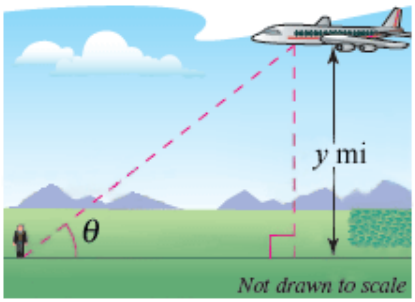
B. $\dfrac{dy}{dx} = \dfrac{-2y}{1 - ye^y}$

C. $\dfrac{dy}{dx} = ye^y - \dfrac{y}{x} - 2y$

D. $\dfrac{dy}{dx} = \dfrac{1 + 2xy}{xye^y}$

E. $\dfrac{dy}{dx} = \dfrac{-2xy - y}{x - xye^y}$

7. An airplane flies at an altitude of $y = 2$ miles towards a point directly over an observer (see figure). The speed of the plane is 500 miles per hour. Find the rate at which the angle of elevation θ is changing when the angle is 60°.



A. $\frac{75}{4}$ radian per hour
B. $\frac{225}{8}$ radian per hour
C. $\frac{125\sqrt{3}}{2}$ radian per hour
D. $\frac{375}{2}$ radian per hour
E. $50\sqrt{3}$ radian per hour

8. Find the critical numbers of $y = x^2e^x$.

A. $x = -2, 1$
B. $x = 0, 2$
C. $x = 0, 1$
D. $x = -2, 2$
E. $x = -2, 0$

9. Given the function

$$f(x) = \frac{8x}{x^2+4},$$

and its derivative,

$$f'(x) = \frac{-8x^2+32}{(x^2+4)^2}.$$

The y values of the absolute maximum and the absolute minimum of $f(x)$ over the closed interval $[-1, 4]$ are respectively:

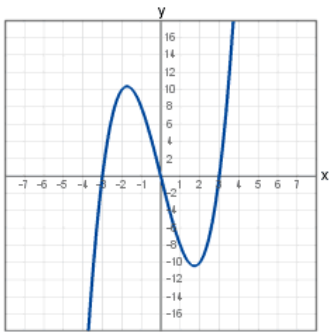
A. $\frac{8}{5}$ and $-\frac{8}{5}$
B. $-\frac{8}{5}$ and -2
C. 2 and $-\frac{8}{5}$
D. $\frac{8}{5}$ and -2
E. 2 and -2

10. Find the open interval where $g(t)$ is increasing.

$$g(t) = -\frac{1}{3}t^3 + \frac{3}{2}t^2$$

A. $(-\infty, 0)$
B. $(0, 3)$
C. $(3, \infty)$
D. $(-\infty, 3)$
E. $(0, \infty)$

11. The graph of the **first derivative** of a function $f(x)$ is shown below. Which of the following statements are true?



(I) $f(x)$ has 2 critical numbers.

(II) On $(-\infty, -3)$, $f(x)$ is increasing.

(III) On $(0, 3)$, $f(x)$ is decreasing.

(IV) A relative maximum occurs at $x = 0$.

A. I and II are true.

B. I and III are true.

C. I and IV are true.

D. II and III are true.

E. III and IV are true.

12. The position function

$$s(t) = t^3 - 2t^2 + t$$

describes the motion of a particle along a line for $t \geq 0$. Choose the correct statement below.

A. The particle is always moving in a positive direction.

B. The particle is always moving in a negative direction.

C. The particle changes from a negative direction to a positive direction at $t = \frac{1}{3}$.

D. The particle changes from a negative direction to a positive direction at $t = 1$.

E. The particle changes from a negative direction to a positive direction at $t = 3$.