1. You must use a #2 pencil on the scantron answer sheet.
2. Fill in your name, your four digit section number, "01" for the Test/Quiz Number, and your student identification number. Make sure to blacken in the appropriate spaces. If you do not know your section number, ask your instructor. Sign your name.
3. There are 15 questions. Blacken in your choice of the correct answer in the spaces provided on the scantron answer sheet. **Only the scantron answer sheet will be graded**.
4. The exam is self-explanatory. Do not ask your instructor any questions about the exam problems.
5. Only one-line calculators (any brand) are allowed. Cell phones and PDA's may not be used as a calculator and must be put away during the exam. NO BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

THE SECOND DERIVATIVE TEST

Suppose f is a function of two variables x and y, and that all the second-order partial derivatives are continuous. Let

$$d = f_{xx}f_{yy} - (f_{xy})^2$$

and suppose (a, b) is a critical point of f.

1. If $d(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f has a relative minimum at (a, b).
2. If $d(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f has a relative maximum at (a, b).
3. If $d(a, b) < 0$, then f has a saddle point at (a, b).
4. If $d(a, b) = 0$, the test is inconclusive.

LAGRANGE EQUATIONS

For the function $f(x, y)$ subject to the constraint $g(x, y) = c$, the Lagrange equations are

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad g(x, y) = c$$

VOLUME & SURFACE AREA

Right Circular Cylinder

$V = \pi r^2 h$

$SA = \begin{cases} 2\pi r^2 + 2\pi rh \\ \pi r^2 + 2\pi rh \end{cases}$

Sphere

$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

Right Circular Cone

$V = \frac{1}{3}\pi r^2 h$

$SA = \pi r\sqrt{r^2 + h^2} + \pi r^2$

1. A company's total profit from selling x thousand units of Product A and y thousand units of Product B is:

$$P(x, y) = 6.3x + 8.4y$$

measured in millions. The quantities produced must satisfy the production possibilities curve:

$$x^2 + y^2 = 144$$

Assuming a maximum exists, how many units of each product should the company produce so that their profit is maximized?

A. $x = 4800$, $y = 7200$

B. $x = 9600$, $y = 7200$

C. $x = 6650$, $y = 10000$

D. $x = 7200$, $y = 9600$

E. $x = 10000$, $y = 6650$

2. Evaluate $\displaystyle\int_1^3 \int_0^2 x^3 y^2 \, dy dx$.

A. 160/3

B. 968/5

C. 484/15

D. 104/3

E. 121/3

3. Use the chain rule to find $\dfrac{dz}{dt}$. Express your answer in terms of x, y, and t.

$$z = \frac{x}{x+y}, \qquad x = e^{t^2}, \qquad y = -e^{t^2}$$

A. $\dfrac{2te^{t^2}(x-y)}{(x+y)^2}$

B. $\dfrac{2te^{t^2}(y-x)}{(x+y)^2}$

C. $\dfrac{2te^{t^2}}{x+y}$

D. $2te^{t^2}$

E. $\dfrac{2te^{t^2}(y-x)}{x+y}$

4. Switch the order of integration of the double integral:

$$\int_{-3}^{0}\int_{x^2}^{9} f(x,y)\,dy\,dx$$

A. $\int_0^3\int_{-\sqrt{y}}^0 f(x,y)\,dx\,dy$

B. $\int_0^9\int_{-\sqrt{y}}^0 f(x,y)\,dx\,dy$

C. $\int_0^9\int_0^{\sqrt{y}} f(x,y)\,dx\,dy$

D. $\int_0^3\int_{-3}^0 f(x,y)\,dx\,dy$

E. $\int_0^9\int_{-3}^0 f(x,y)\,dx\,dy$

5. The profit of a company selling x black shirts and y gold shirts is given by $P(x,y) = 60x + 100y - 150 - 0.1x^2 - 0.2xy - 0.2y^2$ thousand dollars
Find the number of black shirts to supply to maximize the profit.

A. 100

B. 294

C. 200

D. 451

E. 250

6. A winery owner has noticed that her wine receives $P(x, y) = 4x^{2/3}y^{1/2}$ points from a particular judge at a local wine competition if she invests x dollars for grapes per bottle and her winemaker's monthly salary is y hundred dollars. Currently she invests 6 dollars for grapes per bottle and her winemaker's monthly salary is 2500 dollars. Use calculus to estimate the increase in points from the judge when she decides to invest 7 dollars for grapes per bottle and increases her winemaker's monthly salary to 3000 dollars. Round your answer to the nearest tenth.

   A. 8.9

   B. 13.9

   C. 14.1

   D. 102.9

   E. 139.4

7. Find the maximum value of $f(x, y) = x^2 + y^3$ subject to the constraint $4x^2 + y^2 = 64$.

   A. 256

   B. 31.99

   C. 512

   D. 16.29

   E. 509.25

8. In any year, the production of corn can be approximated by

$$C(T, R) = 1896 + 30R + 27RT - T^2 + 172T$$

units per year, where T is average temperature in °F and R is annual rainfall in inches. This year, the average temperature was 52°F and it rained 37.5 inches. If the average temperature is increasing by 0.2°F per year and the annual rainfall is decreasing by 0.015 inches per year, at what rate is the production of corn changing per year? Round your answer to the nearest whole number.

A. -278 units/year

B. -197 units/year

C. 271 units/year

D. 238 units/year

E. 195 units/year

9. Evaluate

$$\int_0^{\pi} \int_0^{y} \csc y \cos x \; dxdy$$

A. 2

B. 1

C. 0

D. π

E. 2π

10. The pressure of an ideal gas, measured in kPa, is related to its volume, V, and temperature, T, by the equation:

$$PV = 0.28T$$

The temperature is measured with an error of 3 Kelvin and the volume is measured with an error of 0.2m^3. If it is know that the actual values are $T = 298$ Kelvin and $V = 5\text{m}^3$, what is the maximum error in the measurement of the pressure? Round your answer to 3 decimal places.

A. 0.499

B. 0.836

C. 1.001

D. 1.024

E. 1.177

11. The cost to make the sides of a rectangular box is \$0.50 per cm^2, and the cost to make the top and bottom is \$0.75 per cm^2. If the volume of the box needs to be 96 cm^3, what is the minimum possible total cost? Round your answer to the nearest dollar.

A. \$36

B. \$48

C. \$72

D. \$82

E. \$108

12. For the function $f(x, y) = (x^3 - 3x)e^y + 2y$, which of the following statements is true?

    A. It has a relative maximum at $(1, 0)$.

    B. It has a relative minimum at $(1, 0)$.

    C. It has a saddle point at $(1, 0)$.

    D. It has two saddle points at $(1, 0)$ and $(-1, 0)$.

    E. It has a relative minimum at $(1, 0)$ and a relative maximum at $(-1, 0)$.

13. You are building a barn, with no floor, in the shape of a rectangular box with a square base. The roof material costs \$15 per m^2, the sides and back material costs \$12 per m^2, and the front material costs \$20 per m^2. The volume of the barn will be 16,000 m^3. What dimensions minimize the total cost?

    A. 17.28 m by 17.28 m by 53.57 m

    B. 31.49 m by 31.49 m by 5.40 m

    C. 20.47 m by 20.47 m by 38.20 m

    D. 28.94 m by 28.94 m by 19.10 m

    E. 31.03 m by 31.03 m by 16.62 m

14. Determine which of the following statements is true regarding the function

$$g(x, y) = 2x^2 - 4xy + y^4 + 13,$$

whose first order partial derivatives are

$$g_x = 4(x - y) \qquad \text{and} \qquad g_y = -4x + 4y^3.$$

A. The function has exactly 1 minimum, exactly 1 saddle point, and no maxima.

B. The function has exactly 1 maximum, exactly 1 saddle point, and no minima.

C. The function has exactly 2 minima, exactly 1 saddle point, and no maxima.

D. The function has exactly 2 maxima, exactly 1 saddle point, and no minima.

E. The function has exactly 2 minima and no maxima or saddle points.

15. In a certain metropolitan area, the population is approximated by the function:

$$P(x, t) = \frac{7280e^{0.5t}}{1 + x}$$

where x is the number of miles from the center of the city, and t is the number of years after the year 2000. What is the average value of the population over the first 8 years within a radius of 5 miles from the city center? Round your answer to the nearest whole number.

A. 34,957

B. 35,609

C. 54,201

D. 139,827

E. 154,470

F14

MA 16020 Exam 3 – Answer Key

Question Number	Green Version Form 01
1	D
2	A
3	C
4	B
5	A
6	B
7	C
8	E
9	D
10	B
11	C
12	C
13	E
14	C
15	A

The exam is worth 120 points

Your score = #correct * 8 points