MA 16021 Exam 3 Memo

Monday, November 17, 2014   6:30pm in Elliott

1. Exam 1 covers sections 6.1, 6.2, 6.3, 6.5, Z1.1, Z4.3, Z4.4, Z11.2 (Lessons 18-27)
2. The exam will consist of 12 multiple choice problems. The exam is machine graded, so there will be no partial credit available.
3. Only a one-line scientific calculator is allowed on any exam. No two-line or graphing calculators are allowed.
4. Since the exam will be machine graded, the only thing that will be graded is the scantron answer sheet. You may keep the exam form. Double check what you are turning in to make sure you have the correct section number and exam form reported and bubbled in. We cannot grade anything on your exam form.
5. There are review problems attached to this memo. It is highly recommended that you review old homework assignments for additional review.
6. Please reread the section on the syllabus regarding the exam.
7. You must use your 10 digit Purdue ID number on the exam scantron. Please double check and make sure that you have filled it in correctly, as this is how it gets uploaded to Blackboard.
8. You must bring your Purdue ID to the exam.
9. If you miss the exam, you need to contact the course coordinator immediately (norris@purdue.edu, MATH 810). Do not wait until the next class session to contact the course coordinator.
10. The exam is self-explanatory. Instructors and proctors are not allowed to interpret any of the questions for any student.
11. Any student that does not have a valid, documented reason for missing an exam may still be allowed to sign up for the make-up exam with a grade penalty. Carelessness in knowing the correct time and place of the exam will not be a valid reason for missing the exam.
12. During the exam, no student is permitted to leave before the first 20 minutes of the exam. No student is allowed to come in and take the exam after the first 20 minutes. If a student shows up more than 20 minutes late, they should speak directly with the course coordinator about the possibility of taking the make-up exam. If they do not have a valid and documentable reason, a grade penalty will be implemented on the make-up exam.

Topics Covered

- Fourier Series (Z11.2)
  - $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{p} + b_n \sin\frac{n\pi x}{p}\right)$ for a function defined on the interval $(-p, p)$

    where $a_0 = \frac{1}{p}\int_{-p}^{p} f(x)dx$ $\quad a_n = \frac{1}{p}\int_{-p}^{p} f(x)\cos\frac{n\pi x}{p}dx$ $\quad b_n = \frac{1}{p}\int_{-p}^{p} f(x)\sin\frac{n\pi x}{p}dx$
  - For any constant function defined by $f(x) = k$ on and interval $(a, b) \subseteq (-p, p)$, the coefficients of the Fourier series can be determined by

    $a_0 = \frac{k}{p}(b-a)$ $\quad a_n = \frac{k}{n\pi}\left(\sin\frac{bn\pi}{p} - \sin\frac{an\pi}{p}\right)$ $\quad b_n = \frac{k}{n\pi}\left(\cos\frac{an\pi}{p} - \cos\frac{bn\pi}{p}\right)$

- If $f(x)$ is linear in the form $f(x) = k + mx$, the coefficients of the Fourier series can be calculated as
  * $a_0 = \frac{1}{p}\left(k(b-a) + \frac{m}{2}\left(b^2 - a^2\right)\right)$
  * $a_n = \frac{k+mb}{n\pi}\sin\frac{bn\pi}{p} - \frac{k+ma}{n\pi}\sin\frac{an\pi}{p} + \frac{mp}{n^2\pi^2}\left(\cos\frac{bn\pi}{p} - \cos\frac{an\pi}{p}\right)$
  * $b_n = \frac{k+ma}{n\pi}\cos\frac{an\pi}{p} - \frac{k+mb}{n\pi}\cos\frac{bn\pi}{p} + \frac{mp}{n^2\pi^2}\left(\sin\frac{bn\pi}{p} - \sin\frac{an\pi}{p}\right)$

- Verifying solutions of differential equations (6.1)
- Separable differential equations (6.2)
  - Solving for general and particular solutions of differential equations by separation of variables: $N(y)dy = M(x)dx$
- Solving first-order linear differential equations by integrating factors: (6.3)
  - Of the form: $\frac{dy}{dx} + P(x)y = Q(x)$
  - Integrating factor: $e^{\int P(x)dx}$
- Applications of differential equations (6.3, 6.4)
  - Growth and decay: $\frac{dy}{dt} = ky$
  - Newton's Law of Cooling: $\frac{dy}{dt} = k(y - T_m)$
  - Families of orthogonal trajectories
  - General rate problems
- Solving higher order homogeneous differential equations (6.5).
- Solving auxiliary equations to determine solutions to higher order homogeneous differential equations. (Z4.3)
  - If m_1, m_2, ...,m_n are solutions of the auxiliary equation, then the general solution of the differential equation is given by $y = C_1e^{m_1x} + C_2e^{m_2x} + ... + C_ne^{m_nx}$
  - If an auxiliary equation has a root $m = a$ repeated n times, then the general solution of the differential equation is given by $y = C_1e^{ax} + C_2xe^{ax} + C_3x^2e^{ax} + ... + C_nx^{n-1}e^{ax}$.
  - If an auxiliary equation has a root $m = a \pm bi$, then the general solution of the differential equation is given by $y = e^{ax}(C_1 \cos bx + C_2 \sin bx)$
- Solving nonhomogeneous differential equations
  - First find the solution to the homogeneous solution, y_c.
  - Use method of undetermined coefficeints to find y_p.

If $f(x)$ is of the form	Use this y_p
e^{kx}	Ae^{kx}
$x^n + x^{n-1} + ...$	$Ax^n + Bx^{n-1} + ...$

Practice Problems

1. Given $f(x) = \begin{cases} 0 & -3 < x < 0 \\ 2 & 0 < x < 3 \end{cases}$, determine a_0, a_n, and b_n of the Fourier series.

2. Given $f(x) = \begin{cases} -5 & -2 < x < 0 \\ 5 & 0 < x < 2 \end{cases}$, determine a_0, a_n, and b_n of the Fourier series.

3. Given $f(x) = \begin{cases} 0 & -7 < x < 0 \\ -4t & 0 < x < 7 \end{cases}$, determine a_0, a_n, and b_n of the Fourier series.

4. Show that $y = e^{4x}$ and $y = e^{-2x}$ are both solutions to the differential equation $\frac{d^2y}{dx^2} - 2\frac{dx}{dy} - 8y = 0$

5. It was determined experimentally that 5% of a quantity of polonium-210 decayed after 20 years. Determine the half-life of Polonium-210.

6. A bacteria culture grows at a rate proportional to the number of bacteria present. If the size of the culture doubles in 2 hours, how long will it take for the size to triple?

7. A body is taken out of a freezer kept at -15^{o}F is plaed in a room whose temperature if 60^{o}F. After 3 minutes the temperature of the bodi has risen to -5^{o}F. Find the time it takes for the temperature of the body to rise to 50^{o}F.

8. The isothermal curves of a metal plate are given by $x^2 - 2y^2 = C$. Find the curves along which the heat flows (the orthogonal trajectories)

9. Find the family of orthogonal trajectories to $y^2 = Cx^5$

10. A 50 gallon tank is full of a solution containing 20 lb of salt. Starting at time $t = 0$, pure water is admitted to the tank at a rate of 5 gal/min, and the well-stirred solution is withdrawn at the same rate.

    (a) How long will it be before the solution will contain half of the original amout of salt?

    (b) Calculate the pounds of water remaining in the tank after 5 minutes.

11. Find the general solution to the following differential equations. Eliminate any logarithms in the solutions, if possible

    (a) $y' + y = 1$

    (b) $1 - x + y^2\frac{dy}{dx} = 0$

    (c) $3x\frac{dy}{dx} = 4y$

    (d) $x\frac{dy}{dx} - 5y = 2x^3$

    (e) $y'' - y' - 2y = 0$

    (f) $\frac{d^2y}{dx^2} - 9y = 0$

    (g) $6y'' + 31y' + 35y = 0$

    (h) $2y'' - 5y' + 8y = 0$

    (i) $3\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$

12. Find the particular solution of the differential equation using the given values.

    (a) $xdy = ydx, y(1) = 2$

    (b) $y^2dy = (1 + x)dx, y(0) = -1$

(c) $y' - 2y = 4, y(0) = 3$

(d) $y'' - 4y = 0, y(0) = 0, y'(0) = 6$

(e) $y'' + 4y = 0, y(0) = 2, y'(0) = 3$

(f) $y'' + 4y' + 4y = 0, y(0) = 4, y'(0) = 5$

13. Use the method of undetermined coefficients to find the particular solution y_p to the following differential equations.

    (a) $y'' - 3y' + 2y = 4e^{3x}$

    (b) $y'' + 3y = 3x^2 + 1$

    (c) $y'' + 5y' + 4y = 4x^4 + x^2 + 2x$

14. Match the following differential equations with a possible graph of its solution

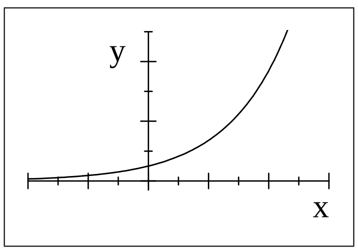
    (a) $y'' - 3y' - 2y = 0$

    (b) $y'' + y = 0$

    (c) $y'' + y' + 3y = 0$

    (d) $y'' + 4y' + 5y = 0$

    (e) $y'' - 2y' + 8y = 0$



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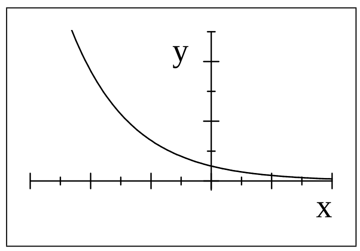

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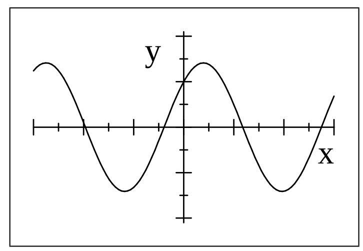

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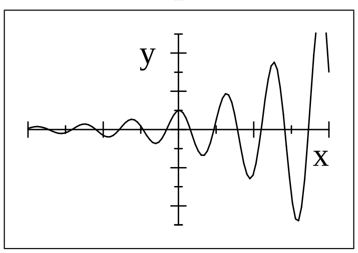

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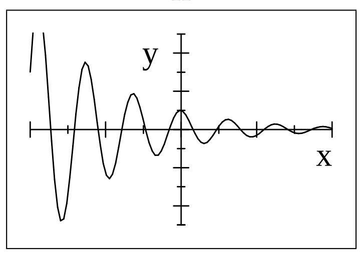

V

Answers

1. $a_0 = 2 \qquad a_n = 0 \qquad b_n = \frac{2}{n\pi}\left(1 - (-1)^n\right)$

2. $a_0 = 0 \qquad a_n = 0 \qquad b_n = \frac{10}{n\pi}\left(1 - (-1)^n\right)$

3. $a_0 = 14 \qquad a_n = \frac{-28}{n\pi}\left((-1)^n - 1\right) \qquad b_n = \frac{28}{n\pi}(-1)^n$

4. (check)

5. 270 years

6. 3.17 hours

7. 42.4 minutes

8. $x^2 y = C$

9. $2x^2 + 5y^2 = C$

10. (a) 6.93 minutes (b) 12.13 lb

11. (a) $y = 1 + Ce^{-x}$

    (b) $y^2 - 3(1-x)^2 = C$

    (c) $\dfrac{y^3}{x^4} = C$

    (d) $y = -x^3 + Cx^5$

    (e) $y = C_1 e^{2x} + C_2 e^{-x}$

    (f) $y = C_1 e^{-3x} + C_2 e^{3x}$

    (g) $y = e^{-\frac{31}{12}x}\left(C_1 \cos\left(\frac{11}{12}x\right) + C_2 \sin\left(\frac{11}{12}x\right)\right)$

    (h) $y = e^{\frac{5}{4}x}\left(C_1 \cos\left(\frac{\sqrt{39}}{4}x\right) + C_2 \sin\left(\frac{\sqrt{39}}{4}x\right)\right)$

    (i) $y = C_1 e^{x} + C_2 e^{3x}$

12. (a) $y = 2x$

    (b) $2y^3 = 6x + 3x^2 - 2$

    (c) $y = -2 + 5e^{2x}$

    (d) $y = -\frac{3}{2}e^{-2x} + \frac{3}{2}e^{2x}$

    (e) $y = \frac{3}{2}\cos 2x + 2\sin 2x$

    (f) $y = 4e^{-2x} - 3xe^{-2x}$

13. (a) $y_p = 2e^{3x}$

    (b) $y_p = x^2 - \frac{1}{3}$

    (c) $y_p = x^4 - 5x^3 + 16x^2 - 32x + 32$

14. (a) I (b) III (c) V (d) II (e) IV