MA 16021 Final Exam Memo and Review
Wednesday, December 17, 2014   3:30pm in WTHR 172

1. PLEASE NOTE THAT THE FINAL EXAM IS IN WTHR 172, A DIFFERENT LOCATION THAN THE MIDTERMS. It is a 2 hours exam that starts at 3:30pm. Please plan to arrive no later than 3:15pm.

2. The final exam is comprehensive. It consists of 20 questions broken down as following:

   (a) 12 questions will come from the first three midterms (literally). There will be 4 questions from Exam 1, 4 questions from Exam 2, and 4 questions from exam 3. The basic concepts will remain the same, but the numbers or formulas will be changed.

   (b) 8 questions will be from material after the exam 3 (Lessons 28-34 covering Z4.4, Z7.1-7.3).

   (c) The final exam will be "in order" – i.e. question 1-4 are from Exam 1, questions 5-8 from Exam 2, 9-12 from Exam 3, and 13-22 from material after Exam 3.

3. The formulas that will be given on the final exam are posted on Blackboard and the course webpage. The formula sheet will be attached to the end of the exam on the last page. so feel free to tear it off for use on the exam.

4. The exam is multiple choice and is machine graded, so there will be no partial credit available.

5. Only a one-line scientific calculator is allowed on any exam. No two-line or graphing calculators are allowed.

6. Since the exam will be machine graded, the only thing that will be graded is the scantron answer sheet. UNLIKE PREVIOUS EXAMS, YOU MUST ALSO TURN IN YOUR COPY OF THE EXAM FORM WHEN YOU TURN IN YOUR ANSWER SHEET. It is Math Department policy to retain copies of the final exam.

7. There are review problems attached to this memo. It is highly recommended that you review old homework assignments for additional review.

8. Please reread the section on the syllabus regarding the exam.

9. You must use your 10 digit Purdue ID number on the exam scantron. Please double check and make sure that you have filled it in correctly, as this is how it gets uploaded to Blackboard.

10. You must bring your Purdue ID to the exam.

11. If you miss the exam, you need to contact the course coordinator immediately (norris@purdue.edu, MATH 810). Do not wait to contact the course coordinator.

12. The exam is self-explanatory. Instructors and proctors are not allowed to interpret any of the questions for any student.

13. Any student that does not have a valid, documented reason for missing an exam may still be allowed to sign up for the make-up exam with a grade penalty. Carelessness in knowing the correct time and place of the exam will not be a valid reason for missing the exam.

14. During the exam, no student is permitted to leave before the first 20 minutes of the exam. No student is allowed to come in and take the exam after the first 20 minutes. If a student shows up more than 20 minutes late, they should speak directly with the course coordinator about the possibility of taking the make-up exam. If they do not have a valid and documentable reason, a grade penalty will be implemented on the make-up exam.

Topics Covered (since Exam 3)

- Solving nonhomogeneous differential equations, such as $ay'' + by' + y = f(x)$
  - First find the solution to the homogeneous solution, y_c.
  - Use method of undetermined coefficeints to find y_p (if $f(x)$ is not a "duplicate" of a solution to the homogeneous equation.

If $f(x)$ is of the form	Use this y_p
e^{kx}	Ae^{kx}
$\sin kx$ or $\cos kx$	$A\sin kx + B\cos kx$
$x^n + x^{n-1} + ...$	$Ax^n + Bx^{n-1} + ...$

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  - Particular solutions y_p can be linear combinations of the above:
    - Say $f(x) = x^2 + 3e^{2x}$, then $y_p = Ax^2 + Bx + C + De^{2x}$
  - If $f(x)$ is a repeated value in the homogeneous solution, add addtional x's to the y_p to make them unique
    - Examples
      - $y'' + 2y' - 3y = 4e^x$: solution to homogeneous is $y_c = C_1e^{-3x} + C_2e^x$ Since e^x is part of the homogeneous solution, then the particular solution $y_p = Axe^x$
      - $y''+4 = 5\cos 2x$: solution to the homogeneous is $y_c = C_1\cos 2x + C_2\sin 2x$. Since $\cos 2x$ is part of the homogeneous solution, then the particular solution $y_p = Ax\cos 2x + Bx\sin 2x$
- Laplace transforms $\mathcal{L}\{f(t)\}$ (see table on Bb or course webpage – will be on the final exam formula sheet)
- Inverse Laplace transforms $\mathcal{L}^{-1}\{F(s)\}$
- Partial fractions with Laplace transforms
- Solving differential equations using Laplace transforms
  - $\mathcal{L}\{y\} = Y(s)$
  - $\mathcal{L}\{y'\} = sY(s) - y(0)$
  - $\mathcal{L}\{y''\} = s^2Y(s) - sy(0) - y'(0)$

Practice Problems

1. Solve the differential equation.
   (a) $y'' - 6y' + 9y = 9x$
   (b) $y'' + 9y = 9e^{3x}$, when $x = 0, y = 1, Dy = 3/2$
   (c) $y'' + y = 6\cos 2x$, when $x = 0, y = 3, Dy = 1$
   (d) $y'' - y = -16\sin x$, when $x = 0, y = 0, Dy = 1$
   (e) $y'' - 2y' + 5y = 5\sin x$, when $x = 0, y = -2, Dy = 0$
2. Find y_p.
   (a) $y'' - 6y = 5\cos x$
   (b) $y'' + 3y = 2x + \cos x$
   (c) $y'' - y' - 3 = 6xe^x - 4\cos x$

3. Find the Laplace transform $\mathcal{L}\{f(t)\}$ of the functions.

   (a) $f(t) = 8e^{-2t}$
   (b) $f(t) = 3 + t^2 - 3\sin 4t$
   (c) $f(t) = t^3e^{-2t} + \sin t$
   (d) $f(t) = 3e^{5t}\cos 2t$

4. Find the inverse Laplace transform $\mathcal{L}^{-1}\{F(s)\}$ of the functions.

   (a) $F(s) = \dfrac{2s}{s^2+7}$
   (b) $F(s) = \dfrac{1}{(s+2)^3}$
   (c) $F(s) = \dfrac{1}{s^2(s^2+1)}$
   (d) $F(s) = \dfrac{s}{s^2+2s-3}$
   (e) $F(s) = \dfrac{s+1}{s^2+2s+5}$
   (f) $F(s) = \dfrac{2s+4}{(s^2+2)^2+4}$

5. Solve the differential equation by the method of Laplace transforms.

   (a) $y' + 3y = 0, y(0) = 2$
   (b) $y' + 2y = e^{2t}, y(0) = 0$
   (c) $y'' + 4y = \sin 2t, y(0) = -2, y'(0) = 0$
   (d) $y'' + 6y' + 13y = 0, y(0) = 1, y'(0) = -2$
   (e) $y'' - 4y = 3\cos t, y(0) = y'(0) = 0$
   (f) $y'' - 4y' + 5y = 4e^t, y(0) = 1, y'(0) = 0$

Answers:

1. (a) $y = c_1e^{3x} + c_2xe^{3x} + x + \frac{2}{3}$ (b) $y = \frac{1}{2}\cos 3x + \frac{1}{2}e^{3x}$
   (c) $y = 5\cos x + \sin x - 2\cos 2x$ (d) $y = \frac{7}{2}e^{-x} + \frac{7}{2}e^{x} + 8\sin x$
   (e) $y = e^x\left(\frac{-5}{2}\cos 2x + \frac{3}{4}\sin 2x\right) + \frac{1}{2}\cos x + \sin x$
2. (a) $y_p = -\frac{5}{7}\cos x$ (b) $y_p = \frac{3}{2}x + \frac{1}{2}\cos x$
   (c) $y_p = -2xe^x - \frac{2}{3}e^x + \frac{16}{17}\cos x + \frac{4}{17}\sin x$
4. (a) $\dfrac{8}{s+2}$ (b) $\dfrac{3}{s} + \dfrac{2}{s^3} - \dfrac{12}{s^2+16}$ (c) $\dfrac{6}{(s+2)^4} + \dfrac{1}{s^2+1}$ (d) $\dfrac{3s-15}{(s-5)^2+4}$
5. (a) $2\cos\sqrt{7}t$ (b) $\frac{1}{2}t^2e^{-2t}$ (c) $t - \sin t$ (d) $\frac{1}{4}e^t + \frac{3}{4}e^{-3t}$
   (e) $(\cos 2t)\,e^{-t}$ (f) $2e^{-2t}\cos 2t$
6. (a) $y = 2e^{-3t}$ (b) $y = \frac{1}{4}e^{2t} - \frac{1}{4}e^{-2t}$
   (c) $y = \frac{1}{8}\sin 2t - 2\cos 2t - \frac{1}{4}t\cos 2t$ (d) $y = e^{-3t}\left(\cos 2t + \frac{1}{2}\sin 2t\right)$
   (e) $y = \frac{3}{10}e^{2t} + \frac{3}{10}e^{-2t} - \frac{3}{5}\cos t$ (f) $y = 2e^t - e^{2t}\cos t$