34. RADIOACTIVE DECAY A sample of radium decays at a rate that is proportional to its size.

35. TEMPERATURE CHANGE The rate at which the temperature of an object changes is proportional to the difference between its own temperature and the temperature of the surrounding medium.

36. DISSOLUTION OF SUGAR After being placed in a container of water, sugar dissolves at a rate proportional to the amount of undissolved sugar remaining in the container.

37. RECALL FROM MEMORY When a person is asked to recall a set of facts, the rate at which the facts are recalled is proportional to the number of relevant facts in the person's memory that have not yet been recalled.

38. MARKET SHARE The rate at which a new product is replacing an old, obsolete product is jointly proportional to the market share of the new product and that of the old product. [*Hint:* Express market share as a percentage.]

39. CORRUPTION IN GOVERNMENT The rate at which people are implicated in a government scandal is jointly proportional to the number of people already implicated and the number of people involved who have not yet been implicated.

40. THE SPREAD OF A RUMOR The rate at which a rumor spreads through a community is jointly proportional to the number of people in the community who have heard the rumor and the number who have not.

41. Verify that the function $y = Ce^{kx}$ is a solution of the differential equation $\dfrac{dy}{dx} = ky$.

42. Verify that the function $Q = B - Ce^{-kt}$ is a solution of the differential equation $\dfrac{dQ}{dt} = k(B - Q)$.

43. Verify that $y = C_1e^x + C_2xe^x$ is a solution of the differential equation $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = 0$.

44. Verify that the function $y = \dfrac{1}{20}x^4 - \dfrac{C_1}{x} + C_2$ is a solution of the differential equation

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x^3.$$

45. OIL PRODUCTION A certain oil well that yields 400 barrels of crude oil per month will run dry in 2 years. The price of crude oil is currently \$130 per barrel and is expected to rise at the constant rate of 4 cents per barrel per month. If the oil is sold as soon as it is extracted from the ground, what will the total future revenue from the well be?

46. AGRICULTURAL PRODUCTION The **Mitscherlich model** for agricultural production specifies that the size $Q(t)$ of a crop changes at a rate proportional to the difference between the maximum possible crop size B and Q; that is,

$$\frac{dQ}{dt} = k(B - Q)$$

a. Solve this equation for $Q(t)$. Express your answer in terms of k and the initial crop size $Q_0 = Q(0)$.

b. A particular crop has a maximum size of 200 bushels per acre. At the start of the growing season ($t = 0$), the crop size is 50 bushels and 1 month later, it is 60 bushels. How large is the crop 3 months later ($t = 3$)?

c. Note that this model is similar to the learning model discussed in Example 8.1.7. Is this just a coincidence or is there some meaningful analogy linking the two situations? Explain.

47. DILUTION A tank holds 200 gallons of brine containing 2 pounds of salt per gallon. Clear water flows into the tank at the rate of 5 gallons per minute, and the mixture, kept uniform by stirring, runs out at the same rate.

a. If $S(t)$ is the amount of salt in solution at time t, then the amount of salt in a typical gallon of solution is given by

$$\frac{\text{Amount of salt}}{\text{Amount of fluid}} = \frac{S(t)}{200}$$

At what rate is salt flowing out of the tank at time t?

b. Write a differential equation for the rate of change of $S(t)$ using the fact that

$$\frac{dS}{dt} = \begin{bmatrix}\text{rate at which}\\ \text{salt enters tank}\end{bmatrix} - \begin{bmatrix}\text{rate at which}\\ \text{salt leaves tank}\end{bmatrix}$$

c. Solve the differential equation in part (b) to obtain $S(t)$. [*Hint:* What is $S(0)$?]

48. THE SPREAD OF AN EPIDEMIC The rate at which an epidemic spreads through a community with 2,000 susceptible residents is jointly