MA 16100 Exam III, Spring 2014, April 17

Name ______________________________

10–digit PUID number ______________________________

Recitation Instructor ______________________________

Recitation Section Number and Time ______________________________

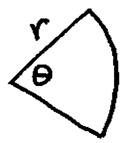
Instructions: MARK TEST NUMBER 01 ON YOUR SCANTRON

1. Do not open this booklet until you are instructed to.
2. Fill in all the information requested above and on the scantron sheet. On the scantron sheet fill in the little circles for your name, section number and PUID.
3. This booklet contains 13 problems, equally weighted.
4. For each problem mark your answer on the scantron sheet and also **circle it in this booklet**.
5. Work only on the pages of this booklet.
6. Books, notes, calculators or any electronic device are not allowed during this test and they should not even be in sight in the exam room. You may not look at anybody else's test, and you may not communicate with anybody else, except, if you have a question, with your instructor.
7. You are not allowed to leave during the first 20 and the last 10 minutes of the exam.
8. When time is called at the end of the exam, put down your writing instruments and remain seated. The TAs will collect the scantrons and the booklets.

Solutions

1. In this picture of a circular sector the area is $\frac{1}{2}r^2\theta$ and the length of the circular arc is $r\theta$, where r is the radius and θ is the opening angle. What is the maximum area among circular sectors whose perimeter is 6?

A. 3.5
B. 4
C. 1.75
D. 3
E. 2.25



$A = \frac{1}{2}r^2\theta \qquad s = r\theta$

Maximize A

$6 = \text{perimeter} = s + 2r = r\theta + 2r$

$r\theta = 6 - 2r, \quad \boxed{\theta = \frac{6}{r} - 2}$

$A(r) = \frac{1}{2}r^2\left(6/r - 2\right) = 3r - r^2$

$A'(r) = 3 - 2r, \quad 3 - 2r = 0, \quad r = 3/2$

$A(3/2) = 3(3/2) - (3/2)^2 = 9/2 - 9/4 = 9/4 = \boxed{2.25}$

2. On what interval(s) is $f(x) = 4x^3 + 3x^2 - 6x$ decreasing?

A. $\left(-2, -\frac{1}{2}\right)$
B. On $\left(-\infty, \frac{3}{2}\right)$ and on $\left(\frac{3}{2}, \infty\right)$
C. On $(-\infty, -1)$ and on $\left(\frac{1}{2}, \infty\right)$
D. $\left(\frac{1}{2}, 1\right)$
E. $\left(-1, \frac{1}{2}\right)$

$f'(x) = 12x^2 + 6x - 6$
$= 6(2x^2 + x - 1)$
$= 6(2x - 1)(x + 1)$

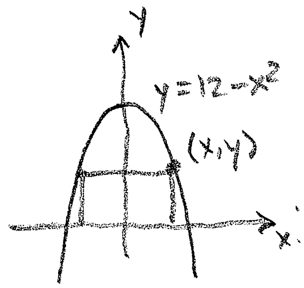
$1/2 \qquad -1$

	(-2)	-1	(0)	1/2	(1)
$f'(x)$	+		−		+
$f(x)$	/		\		/

$(-1, 1/2)$.

3. A rectangle with sides parallel to the axes is inscribed in the region above the x-axis and below the parabola $y = 12 - x^2$. The maximum area of such a rectangle is

A. 48
B. 6
C. 32
D. 12
E. 24



Maximize area A

$A = 2xy \qquad y = 12 - x^2$

$A(x) = 2x(12 - x^2) = 24x - 2x^3, \quad 0 < x < 2.$

$A'(x) = 24 - 6x^2, \qquad 24 - 6x^2 = 0$

$6x^2 = 24$

$x^2 = 4$

$x = \pm 2$

$x = 2$

$A(2) = 24(2) - 2(2)^3$

$= 48 - 16$

$= \boxed{32}.$

4. If $g'(x) = \dfrac{1}{\sqrt{x}} + 2x$ and $g(1) = 0$, then $g(3) =$

A. $1/3$
B. $6 + 2\sqrt{3}$
C. $2 + \sqrt{3}$
D. 0
E. $4 + \sqrt{3}$

$g'(x) = x^{-1/2} + 2x$

$g(x) = 2x^{1/2} + \cancel{2}x^2/\cancel{2} + c$

$0 = g(1) = 2 + 1 + c, \quad c = -3$

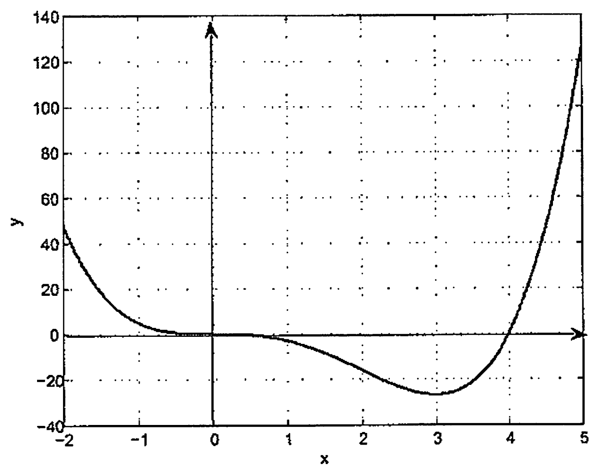
$g(x) = 2x^{1/2} + x^2 - 3$

$g(3) = 2 \cdot 3^{1/2} + 3^2 - 3$

$= \boxed{2\sqrt{3} + 6}$

5. The graph of $y = f(x)$ is shown below. On what interval(s) is $g(x) = \int_{-2}^{x} f(t)dt$ increasing?

$g'(x) = \frac{d}{dx}\int_{-2}^{x} f(t)\,dt = f(x) > 0$ for x in $\boxed{(-2,0)\cup(4,5)}$



A. On $(-2, 2)$ and on $(3, 5)$
B. $(2, 4)$
C. On $(-2, 0)$ and on $(4, 5)$
D. $(0, 4)$
E. $(3, 5)$

6. Find the limit

$$\lim_{x\to\infty} x\tan\left(\frac{1}{x}\right)$$ $(\infty \cdot 0)$ As $x \to \infty$, $\frac{1}{x} \to 0$

$\tan\frac{1}{x} \to 0$

$= \lim_{x\to\infty} \frac{\tan(1/x)}{1/x}$ $\left(\frac{0}{0}\right)$

$\overset{H}{=} \lim_{x\to\infty} \frac{\sec^2(1/x)\cdot \cancel{(-1/x^2)}}{\cancel{(-1/x^2)}}$

$= \sec^2\left(\lim_{x\to\infty} 1/x\right)$

$= \sec^2(0)$

$= \boxed{1}$.

A. 0
B. -1
C. 1
D. $-\infty$
E. ∞

7. Suppose $F(t)$ is a function of t in $(-\infty,\infty)$. Assume that $F'(t)$ and $F''(t)$ are continuous on $(-\infty,\infty)$. Let c be a real number. Which is true?

I. If F has a local maximum at c, then $F'(c)=0$.
II. If $F'(c)=0$, then F has a local maximum or a local minimum at c.
III. If $F'(c)=0$ and $F''(c)>0$, then F has a local minimum at c.

A. Only I and II
B. Only I
C. Only I and III
D. Only II
E. Only III

I is true. If F has a local max at c then $F'(c)=0$ or $F'(c)$ does not exist. Since F' is continuous $F'(c)$ exists. Therefore $F'(c)=0$.

II is false. $F(t)=t^3$ has no local maximum or minimum but $F'(0)=0$.

III is true. Second derivative test

$\boxed{\text{Only I and III}}$

8. Which of the following pictures most resembles the graph $y = 3x - x^3$ between $x = -\frac{1}{2}$ and $x = \frac{1}{2}$?

(A)

(B)

(C)

(D)

(E)

$y' = 3 - 3x^2 = 3(1-x^2) = 3(1-x)(1+x)$

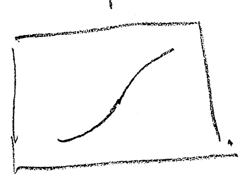
$1 \quad -1$

	-1	0	1
y'		+	
y		/	

$y'' = -6x$

0

	-1	0	1
y''	+		−
y	CU		CD



9. Evaluate the integral

$$\int_0^1 x(2x+1)dx$$

A. 1/2
B. 7/6
C. 4/3
D. 2/3
E. 5/6

$= \int_0^1 2x^2 + x \; dx$

$= \frac{2x^3}{3} + \frac{x^2}{2} \Big|_0^1$

$= \frac{2}{3} + \frac{1}{2}$

$= \frac{4}{6} + \frac{3}{6}$

$= \boxed{\frac{7}{6}}$

10. If $f(1) = 2$, $f'(x) \geq 1$, for $1 \leq x \leq 4$, what is the least value $f(4)$ can be?

A. 9
B. 5
C. 8
D. 6
E. 7

$1 \leq f'(x) = \frac{f(4) - f(1)}{4-1} = \frac{f(4)-2}{3}$ by MVT

$f(4) - 2 \geq 3$

$f(4) \geq \boxed{5}$

11. Which of the following pictures most resembles the graph $y = x + \dfrac{1}{x}$ between $x = -3$ and $x = -2$?

(A)

(B)

(C)

(D)

(E)

$$y' = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$$

± 1

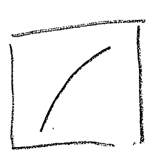
0

	(−2)	−1		0		1	
y'	⊕						
y	↗						

$$y'' = \frac{2}{x^3}$$

0

	(−1)	0	
y''	⊖		
y	CD.		



12. Find the absolute maximum of $f(x) = x^3 - 3x + 1$ on $[0, 2]$.

A. 4
B. 6
C. 3
D. 5
E. 2

① $f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x-1)(x+1)$

$1 \quad -1$. (Ignore -1.)

②

x	$f(x)$
0	1
1	-1
2	$\boxed{3}$

13. Find the derivative of the function

$$f(x) = \int_{2\pi}^{e^x} \sqrt{1 + \cos(t)}\, dt$$

A. $e^x\sqrt{1 + \cos(e^x)}$
B. $\dfrac{-e^x}{2\sqrt{1 + e^x}}$
C. $e^x\sqrt{1 - \sin(e^x)}$
D. $2e^x\sqrt{1 - \sin(e^x)}$
E. $2e^x\sqrt{1 + \cos(e^x)}$

$$f'(x) = \frac{d}{dx}\int_{2\pi}^{e^x} \sqrt{1 + \cos(t)}\, dt = \boxed{\sqrt{1 + \cos e^x} \cdot e^x}$$