MA 23100 - Practice Exam 1

1. Solve $3x^2 - 4x = 1$.

   A. $x = \frac{1}{3},\ x = 1$
   B. $x = \frac{4}{3},\ x = -1$
   C. $x = \frac{2 \pm \sqrt{7}}{3}$
   D. $x = \frac{2 \pm \sqrt{14}}{6}$
   E. $x = \frac{1 \pm 2\sqrt{14}}{3}$

2. Find the domain of $f(x) = \dfrac{x^2 + 5x + 6}{x^2 + 4x + 3}$.

   A. $\{x \mid x$ is a real number and $x \neq -3$ and $x \neq -2\}$
   B. $\{x \mid x$ is a real number and $x \neq -1\}$
   C. $\{x \mid x$ is a real number and $x \neq -2\}$
   D. $\{x \mid x$ is a real number and $x \neq -3$ and $x \neq -1\}$
   E. $\{x \mid x$ is a real number and $0 < x < 3\}$

3. $\cot^2 x - \cos^2 x$ simplifies to

   A. $\cos^2 x \sin^2 x$
   B. $\cot^2 x \cos^2 x$
   C. 1
   D. $\sin^2 x$
   E. $\sin^4 x$

4. Give a periodic function of the form $f(x) = a \sin(bx) + k$, where $f(x)$ has a average value of 9, minimum value of 3, and a period of 8.

   A. $f(x) = 6 \sin(8x) + 9$
   B. $f(x) = 9 \sin(8x) + 6$
   C. $f(x) = 3 \sin(8x) + 9$
   D. $f(x) = 9 \sin\left(\frac{\pi x}{4}\right) + 6$
   E. $f(x) = 6 \sin\left(\frac{\pi x}{4}\right) + 9$

5. If the temperature remains constant, the pressure of an enclosed gas is inversely proportional to the volume. The pressure of helium within a spherical balloon of radius 9 inches is 20 lb/in^2.

   The volume of a sphere with radius r is given by $V = \frac{4}{3}\pi r^3$. If the radius of the balloon increases to 12 inches what is the pressure of the gas?

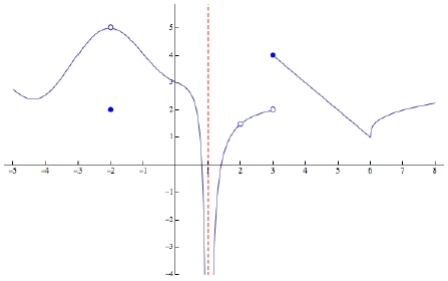
   A. 26.67 lb/in^2
   B. 8.438 lb/in^2
   C. 0.119 lb/in^2
   D. 21.917 lb/in^2
   E. 15 lb/in^2

6. Given that the solutions to $2x^2 + x - 1 = 0$ are $x = \frac{1}{2}$ and $x = -1$, solve

$$2\sin^2(3x) + \sin(3x) - 1 = 0.$$

   A. $x = \frac{\pi}{18} + \frac{2\pi n}{3}, x = \frac{5\pi}{18} + \frac{2\pi n}{3}, x = \frac{\pi}{2} + \frac{2\pi n}{3}$
   B. $x = \frac{\pi}{6} + \frac{2\pi n}{3}, x = \frac{5\pi}{6} + \frac{2\pi n}{3}, x = \frac{\pi}{2} + \frac{2\pi n}{3}$
   C. $x = \frac{\pi}{6} + 2\pi n, x = \frac{5\pi}{6} + 2\pi n, x = \frac{3\pi}{2} + 2\pi n$
   D. $x = \frac{\pi}{18} + 2\pi n, x = \frac{5\pi}{18} + 2\pi n, x = \frac{\pi}{2} + 2\pi n$
   E. $x = \frac{\pi}{9} + 2\pi n, x = \frac{2\pi}{9} + 2\pi n, x = \frac{3\pi}{2} + 2\pi n$

Consider the plot of $f(x)$ below for the next two problems.



7. Find the number of correct statements.

   I. There are four discontinuities shown above.
   II. It is possible to define $f(3)$ so that $f(x)$ is continuous at $x = 3$.
   III. $f(x)$ is discontinuous on the interval $(4, 7)$.
   IV. It is possible to define $f(2)$ so that $f(x)$ is continuous at $x = 2$.

   A. There are zero correct statements
   B. There is only one correct statement
   C. There are only two correct statements
   D. There are only three correct statements
   E. All statements are correct

8. In the plot above, if $a = \lim_{x \to -2} f(x)$ and $b = \lim_{x \to 3^-} f(x)$, find ab.

   A. 8
   B. 4
   C. 20
   D. 10
   E. 15

9. Evaluate $\lim_{x \to 4} \dfrac{(x-4)^3}{(x^2+3x-28)^3}$.

   A. $\frac{1}{1330}$
   B. $\frac{1}{1331}$
   C. $\frac{1}{1332}$
   D. $\frac{1}{1333}$
   E. $\frac{1}{1334}$

10. Find the simplified difference quotient of $f(x) = 3x^2 - 5x + 7$.

    A. $3h^2 + 6hx - 5h - 10x + 14$
    B. $3h - 5$
    C. $6x - 5 + 3h$
    D. $6x - 5 + 3h^2$
    E. $3h + 5$

11. Given the derivative of $f(x) = x^2 - \dfrac{1}{x}$ is $f'(x) = 2x + \dfrac{1}{x^2}$, find the equation of the tangent line to $f(x) = x^2 - \dfrac{1}{x}$ at the point $x = 1$.

    A. $y = 3x - 3$
    B. $y = 3x + 1$
    C. $y = x - 1$
    D. $y = 2x - 4$
    E. $y = 2x - 2$

12. If $f(x) = \left(x - x^2\right)^2 - 3\sin x$, find $f'(\pi)$.

    A. $4\pi^3 - 4\pi^2 - 3$
    B. $-2\pi^2 + 2\pi - 3$
    C. $-2\pi^2 + 2\pi + 3$
    D. $4\pi^3 - 6\pi^2 + 2\pi - 3$
    E. $4\pi^3 - 6\pi^2 + 2\pi + 3$