MA 23200 - Practice Exam 2

1. Given that $f(x,y) = \sqrt{x^2 - y^3}$, compute $f_y(3,-2)$.

   A. $\frac{-6\sqrt{17}}{17}$

   B. $\frac{\sqrt{17}}{34}$

   C. $\frac{-3\sqrt{17}}{17}$

   D. $\frac{-12\sqrt{17}}{17}$

   E. $\frac{3\sqrt{17}}{34}$

2. Let $h(x,y) = \cos^3\left(\frac{y}{x}\right)$. Calculate $h_x(3,\pi)$.

   A. $-\frac{\pi}{12}$

   B. $\frac{\pi}{12}\ln 3$

   C. $-\frac{3\pi}{24}$

   D. $\frac{9\pi}{8}\ln 3$

   E. $\frac{\pi\sqrt{3}}{24}$

3. For $f(x, y) = 1 + x\ln(xy - 5)$, find $f_{xy}(2, 3)$.

   A. -5

   B. -1

   C. -2

   D. -8

   E. -4

4. The first order partial derivatives of $f(x, y) = 2x^2y + xy^2$ are

$$f_x = 4xy + y^2 \qquad\qquad f_y = 2x^2 + 2xy.$$

   Use linearization at the point $(1, 2)$ to estimate $f(0.93, 2.08)$.

   A. 7.65

   B. 7.64

   C. 7.63

   D. 7.62

   E. 7.61

5. Choose the correct statement below about the critical points of

$$f(x,y) = xe^{x+\frac{1}{2}y^4-4y^2}.$$

.

A. $f(x,y)$ has no critical points.

B. $f(x,y)$ has exactly one critical point.

C. $f(x,y)$ has exactly two critical points.

D. $f(x,y)$ has exactly three critical points.

E. $f(x,y)$ has exactly four critical points.

6. The function $f(x,y) = 6x^2 - x^3 - 3xy - 15x + \frac{1}{2}y^2 + 11y$ has partial derivatives

$$f_x(x,y) = 12x - 3x^2 - 3y - 15, \qquad f_y(x,y) = y - 3x + 11$$

and critical points

$$(3,-2), \quad (-2,-17)$$

(There are no other critical points). Which statement best describes these critical points?

A. Both points are saddle points.

B. $(3,-2)$ is a saddle point and $(-2,-17)$ is a relative maximum.

C. $(3,-2)$ is a saddle point and $(-2,-17)$ is a relative minimum.

D. $(3,-2)$ is a relative minimum and $(-2,-17)$ is a relative maximum.

E. $(3,-2)$ is a relative maximum and $(-2,-17)$ is a relative minimum.

7. A company estimates that the total cost of a project, in millions of dollars, is a function C given by

$$C(t, n) = 5t^2 + 3n^2 - 84n - 2tn + 748$$

where t is the number of years planned for the project, and n is the number of employees assigned to the project (in hundreds). Find the minimum possible cost of the project.

A. $94 million

B. $182 million

C. $225 million

D. $321 mllion

E. $118 million

8. Find the slope of the least-squares regression line for the following set of points:

$$(3, 3), (4, 11), (5, 13), (6, 20).$$

A. 4.4

B. 4.7

C. 4.9

D. 5.3

E. 5.7

9. Evaluate the double integral:

$$\int_0^{\pi}\int_0^{1} y(y+1)\cos(xy)\,dx\,dy.$$

A. $-\pi+1$

B. $-\pi$

C. π

D. $\pi+2$

E. $\pi+1$

10. The region G is bounded by $y = 2 - x$, $y = x - 2$ and the y-axis. Evaluate the double integral

$$\iint_G (x+2y)\,dy\,dx.$$

A. $-\frac{8}{3}$

B. $\frac{8}{3}$

C. $-\frac{40}{3}$

D. $\frac{40}{3}$

E. $-\frac{32}{3}$

11. The region G is bounded by $y = -\tan x$, $y = \tan x$, and $x = \frac{\pi}{3}$. Evaluate the double integral

$$\iint_G \sec^3 x \, dy \, dx$$

.

A. $\frac{16}{3}$

B. $\frac{7}{3}$

C. $\frac{14}{3}$

D. $\frac{16\sqrt{3}}{27}$

E. $\frac{16\sqrt{3}-18}{27}$

12. In psychology, the Weber-Fechner model of stimulus-response asserts that the rate of change $\frac{\mathrm{d}R}{\mathrm{d}S}$ of the reaction R with respect to a stimulus S is inversely proportional to the stimulus. That is,

$$\frac{\mathrm{d}R}{\mathrm{d}S} = \frac{k}{S},$$

where k is some positive constant. We also assume that $S > 0$. Let S_0 be the detection threshold value, so that $R(S_0) = 0$. Find $R(S_1)$.

A. 0

B. $k \ln \frac{S_1}{S_0}$

C. $k \ln \frac{S_0}{S_1}$

D. $k \ln(S_1 - S_0)$

D. $k \ln(S_0 - S_1)$

Formulas

1. Linearization for a function $f(x, y)$ of two variables, the linearization at the point (a, b) is given by:

$$f(x, y) \approx f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b).$$

2. D-Test to find the relative maximum and minimum values of f:

   (1) Find f_x, f_y, f_{xx}, f_{yy} and f_{xy}.

   (2) Solve $f_x(x, y) = 0$ and $f_y(x, y) = 0$.

   (3) Evaluate $D = f_{xx}f_{yy} - [f_{xy}]^2$ at each point (a, b) found in Step 2.

      (a) If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f has a relative maximum at (a, b).

      (b) If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f has a relative minimum at (a, b).

      (c) If $D(a, b) < 0$, then f has a saddle point at (a, b).

      (a) If $D(a, b) = 0$, then the test is inconclusive. You will have to do something else to determine what is happening at that point.

3. Method of Least Squares.
   The line of least squares regression for the n points (c_1, d_1), (c_2, d_2),...,(c_n, d_n) is given by:

$$y - \bar{y} = m(x - \bar{x})$$

   where,

$$\bar{x} = \frac{\sum_{i=1}^{n} c_i}{n}, \qquad \bar{y} = \frac{\sum_{i=1}^{n} d_i}{n}, \qquad m = \frac{\sum_{i=1}^{n}(c_i - \bar{x})(d_i - \bar{y})}{\sum_{i=1}^{n}(c_i - \bar{x})^2}.$$